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Kinetic theory of drag on objects in nearly free molecular flow

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HIGHLIGHTS

- Collision integrals for drag coefficient.
- Drag of disc.
- Drag of sphere.

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ABSTRACT

Using an analogy between the density expansion of the transport coefficients of moderately dense gases and the inverse-Knudsen-number expansion of the drag on objects in nearly free molecular flows, we formulate the collision integrals that determine the first correction term to the free-molecular drag limit. We then show how the procedure can be applied to calculate the drag coefficients of an oriented disc and a sphere as a function of the speed ratio.

1. Introduction

The drag force on a solid object moving in a rarefied gas has been and remains a subject of great technological interest [1–3]. An important parameter in the theory of rarefied gas flows is the Knudsen number Kn, which is the ratio of the mean free path l of the molecules and a length R that characterizes the size of the object. The limit $Kn \to \infty$ corresponds to the free-molecular flow regime in which the drag is solely determined by the number of individual gas molecules striking the object and the collision dynamics. The limit $Kn \to 0$ corresponds to the continuum regime in which one needs to solve the full nonlinear Boltzmann equation subject to the appropriate boundary conditions. A proper theory for the drag force at arbitrary Knudsen numbers is an interesting and important challenge [3–9]. In the absence of reliable theoretical predictions, one often resorts to empirical correlations [10].

Here we consider the drag coefficient of objects in nearly free molecular flow, where the Knudsen number is large but not infinite. As pointed out by Dorfman et al. [11–13], there is a close similarity between the density expansion of transport coefficients of moderately dense gases and an expansion of the drag coefficient of objects around the free-molecular-flow limit in terms of the inverse Knudsen number Kn^{-1}. For instance, the viscosity μ of a moderately dense gas has an expansion

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in terms of the density n of the form [14]

$$\mu = \mu_0 + \mu_1 n + \mu_2' n^2 \log n + \mu_2'' n^2 + \cdots. \tag{1.1}$$

In this expansion the viscosity μ_0 in the dilute-gas limit is determined by uncorrelated binary collisions between the gas molecules, the coefficient μ_1 by correlated collision sequences among three molecules, and μ_2' by correlated collision sequences among four molecules. The drag coefficient C_D of an object is defined as

$$C_D = \frac{F}{U_K}, \tag{1.2}$$

where F is the magnitude of the force exerted on the object and U_K the incident kinetic energy. In the nearly free-molecular flow regime this drag coefficient has an expansion of the form

$$C_D = C_0 + C_1 Kn^{-1} + C_2' Kn^{-2} \log Kn^{-1} + C_2'' Kn^{-2} + \cdots. \tag{1.3}$$

The expressions for the coefficients in the expansion of the drag coefficient can be obtained by applying a Knudsen-number iteration to the solution of the Boltzmann equation in the presence of the object [12,13]. One then finds that the expressions for the coefficients in (1.3) are related to the same dynamical events that determine the coefficients in (1.1), but with one of the molecules replaced with the foreign object. The expansion (1.3) is valid when the mean free path of the molecule is large compared to the size of the object in all three dimensions. When the mean free path is large compared to the size of the object in two dimensions, but not in the third dimension, for example, in the case of the drag on a cylinder or on a strip, a logarithmic dependence on the inverse Knudsen number already appears in the first correction to the free-molecular-flow limit, so that the expansion for the drag coefficient becomes [13,15–17]

$$C_D = C_0 + C_1' Kn^{-1} \log Kn^{-1} + C_2'' Kn^{-1} + \cdots, \tag{1.4}$$

again in analogy to the logarithmic density expansion of the transport coefficients of a two-dimensional gas [18,19]. In addition it should be noted that the coefficient C_1'' of the term linear in Kn^{-1} in (1.4) depends logarithmically on the flow velocity, so that expansion (1.4) is only valid for finite values of the flow velocity [13]. There exists a similar analogy between the density expansion of the transport properties of moderately dense gases and the density expansion of the friction coefficient of a Brownian particle [20].

The present paper will only deal with the drag on objects whose size is small in all directions compared to the mean free path, so that the first correction to the free-molecular drag force is linear in Kn^{-1} in accordance with (1.3). The specific purpose of the present paper is to formulate the collision integrals that determine the amplitude C_1 of the first inverse-Knudsen-number correction to the free-molecular flow drag and then show how they can be evaluated to determine the magnitude of this correction for a disc oriented perpendicular to the flow and for a sphere as representative examples. In principle, our method of calculating the drag coefficient from collision integrals can be applied to objects of any shape and for any interactions of the gas molecules with the solid surface of the object. In practice we shall introduce a number of simplifying approximations:

- The molecules that strike the object do not stick to it, but are re-emitted after a time short compared to the mean free time of the molecules.
- The molecules are re-emitted diffusively with a temperature T corresponding to the temperature of the object, which is assumed to be the same as the temperature of the molecules in the gas stream.
- The molecules in the gas stream have a mass m and interacts with short-ranged repulsive forces of range σ.
- The solid object is convex (non-concave), so that a molecule emitted from the surface of the object cannot strike the object unless it first collides with another molecule.

The drag force on the object not only depends on the Knudsen number, but also on the Mach number M, defined as the ratio of the magnitude of the flow velocity V and the sound velocity. Instead of the Mach number, we shall use the speed ratio, which is the ratio of V and the thermal molecular velocity:

$$S = V\,(m/2k_B T)^{1/2}, \tag{1.5}$$

where k_B is Boltzmann's constant. The speed ratio is directly proportional to the Mach number as $S = M\,(\gamma/2)^{1/2}$, where γ is the ratio of the isobaric and isochoric heat capacities [1].

As in the case of the collision integrals appearing in the density expansion of the transport coefficients of moderately dense gases [21,22], we find it convenient to represent the molecular collisions and the interactions of the molecules with the solid object by binary collision operators to be defined in Section 2. Using then an expansion in terms of these binary-collision operators, we formulate in Section 3 the specific collision sequences that contribute to the first correction of the drag force beyond the free-molecular flow limit. The explicit expressions for the relevant collision integrals are presented in Section 4. As representative examples, we evaluate these collision integrals for the drag coefficient of a disc in Section 5 and for a sphere in Section 6. A brief summary of our results is presented in Section 7.

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