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Critical thresholds for scale-free networks against cascading failures

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HIGHLIGHTS

- A load redistribution model under single node loss in networks is proposed.
- Thresholds can determine if a network is susceptible to breakdown, or is immune.
- Changing trends for two critical thresholds are in an opposite way with τ and β.
- γ can affect the cascading process of scale-free network within LLPSR and GLPSR.

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ABSTRACT

We explore the critical thresholds of scale-free networks against cascading failures with a tunable load redistribution model which can tune the load redistribution range and heterogeneity of the broken node. Research suggests that the critical behavior belongs to the universality class of global load preferential sharing rule (GLPSR) and local load preferential sharing rule (LLPSR) in networks. Networks collapse completely when $\alpha < \alpha_{c1}$ and are immune to single failure when $\alpha > \alpha_{c2}$. The changing trends for the critical thresholds of α_{c1} and α_{c2} are in an opposite way with initial load distribution coefficient and redistribution heterogeneity coefficient. It means networks may show different properties in the middle ground between total robustness and total collapse. Another striking finding is that the decrease of the exponent γ ($\gamma > 1$) of scale-free networks would make the system stronger against cascading failure within LLPSR and GLPSR.

1. Introduction

In recent years, major disasters occur frequently on the Internet, power grids as well as other infrastructure networks, attracting more and more experts in various fields to study these catastrophic events [1–8]. From the current study results, most of these disasters can be considered, to some extent, to be triggered by minor events. The network in the real world should carry materials, energy, information, date, or some form of loads in its evolutionary process. All these networks have some dynamic behaviors, such as the network topologies have to undertake all forms of flows. Hence, the overload mechanism of cascading failure (OMCF) is a simplified model that is often used to represent real dynamics. It has been successfully

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used to probe the behavior of catastrophic accidents on networks. This paper focuses on some of the basic properties of this dynamic behavior.

The OMCF was introduced into complex networks by Motter and Lai [9] with an ML model, in which the initial load F_i of node i is assumed to be its betweenness, its capacity C_i is proportional to its initial load: $C_i = (1+\alpha)F_i$, and the damage caused by a cascade is quantified in terms of the size of the largest connected component after the cascade relative to its original size. Here, the constant α is a capacity coefficient representing the ability of the node to afford extra load, which actually characterizes the fault tolerance of networks. Networks can handle better with the cascade if the value of α is designed higher. Intuitively, for a given network, there always are two thresholds of α, where cascading failures can cause the network to disintegrate almost entirely when $\alpha < \alpha_{c1}$, and the network is immune to cascading breakdown when $\alpha > \alpha_{c2}$. α_{c1} and α_{c2} respectively are the first and the second phase-transition points. The simulation results in Ref. [9] show that the two phase-transition points do exist. Zhao et al. uncovered the phase-transition phenomenon of scale-free networks with the ML model, and the analytic formulas of α_{c1} and α_{c2} have been determined [10,11]. The load redistribution mechanism of the ML model is based on the concept of betweenness, which means that the betweenness of the new network formed by removal of one node represents the updated load. The dynamic process of this mechanism is based on a fundamental assumption that all nodes in the network can master the global information to fit the new network instantly. Compared with the global redistribution mechanism of the ML model, Wang and Chen [12] proposed a nearest neighbor load redistribution model, in which the load of the broken node is carried by its nearest neighbors, and Wu et al. [13] examined the impact of the initial load distribution on the cascading process of the system. Wang and Rong [14,15] improved the model by adjusting the preferential weight, in which the degree of the broken node as well as its neighbor's degree is considered. It assumes that the initial load distribution strength be equal to the load redistribution homogeneity parameter. Using the normalized avalanche size to quantify the robustness of the network based on the improved model, an estimate of lower bound of the second phase-transition point is given. The load redistribution mechanism of these neighbor sharing rules is based on a typical concept that the node in the system can only master its neighbor's information.

However, the node in the real system is always an agent, which may not only obtain its neighbor's surrounding information but also it can grasp some global information through certain means. When congestion occurs in the traffic network, the information conveyed by the traffic police in congested intersection and real-time traffic broadcasts would affect the distribution of traffic flow to some extent, which would help the traffic flow intending to cross the congestion road reselect a more appropriate path. In addition, there is a similar situation on the Internet. Once one critical router failed, the control system has to reselect routers to transfer data. The routers redistributed by the control system may not only include its neighbor routers. According to the redistribution rule of real networks always lies between the global preferential rule and the local preferential rule or between even the shared rule and the extremely heterogeneous rule, a new cascading model is proposed based on a tunable load redistribution model. It can tune the load redistribution range and the redistribution heterogeneity of extra load respectively by a redistribution range coefficient and a redistribution heterogeneity coefficient. In fact, this model can reflect the management effect of the network administrator.

In this paper, we investigate a model for load redistribution under single node loss in scale-free networks and characterize the critical thresholds of a robustness parameter that determine if a network is susceptible to total disintegration or is immune. From the physical sense, the thresholds correspond to two types of phase-transition phenomenon on networks. Single threshold is difficult to fully and accurately reflect the network's robustness against cascading failure, as it just means that there is a sharp transition from total robustness to susceptibility to total collapse. In theory, we can easily understand that the overall resilience of the network is improved as both of α_{c1} and α_{c2} decrease. But, do the thresholds decrease as we desire when something about the network changes? Based on the tunable load distribution model, this paper mainly focuses on the evolution mechanism of the critical thresholds on scale-free networks and discusses the properties of the middle ground between total robustness and total susceptibility with an analytical method and numerical simulation.

2. Cascading failure model

For simplicity, assume that initial load F_i of node i be a function of its degree k_i and defined as $F_i = \rho k_i^{\tau}$. Here, ρ is a constant value. τ is a tunable parameter, which controls the strength of the initial load. This assumption of non-dimensional structure load is reasonable and valid when it is hard to determine the real load of the system, e.g., data flow of every node on Internet usually has a certain correlation with its degree. Following the previous ML model, each node i in the network has a capacity coefficient, which is the maximum flow that the node can carry. Since the node capacity on real-life networks is generally limited by cost and technique, it is natural to assume that the capacity C_i of the node i be proportional to its initial load for simplicity: $C_i = (1+\alpha)F_i$ [9,15].

The load of the broken node i will be redistributed to some or all of the intact nodes, which would cause one update of F_j

$$F_j \to F_j' = F_j + \Delta F_j. \tag{1}$$

Assume that the extra load ΔF_j of node j be proportional to the broken load F_i

$$\Delta F_j = F_i \cdot p(l_{ij}, \theta, k_j, \beta). \tag{2}$$

Here, $p(l_{ij}, \theta, k_j, \beta)$ is the preferential probability, l_{ij} represents the distance from node i to node j, if they are 1-order neighbors, then $l_{ij} = 1$. θ and β are redistribution policy parameters respectively to control the redistribution range and