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Statistical and dynamical properties of a dissipative kicked rotator

Q1 Diego F.M. Oliveira *, Edson D. Leonel

UNESP, Univ Estadual Paulista, Departamento de Física, Av.24A 1515, Bela Vista, 13506-900, Rio Claro, SP, Brazil

HIGHLIGHTS

- A problem of relativistic particles in a waveguide.
- Transition from local to global chaos occurs.
- The structure of the phase space changes attractors appear.
- Scaling analysis and analytical arguments reinforcing the scaling exponents obtained.
- Parameter space with infinite families of shrimp-shape structures.

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ABSTRACT

Some dynamical and statistical properties for a conservative as well as the dissipative problem of relativistic particles in a waveguide are considered. For the first time, two different types of dissipation namely: (i) due to viscosity and; (ii) due to inelastic collision (upon the kick) are considered individually and acting together. For the first case, and contrary to what is expected for the original Zaslavsky's relativistic model, we show there is a critical parameter where a transition from local to global chaos occurs. On the other hand, after considering the introduction of dissipation also on the kick, the structure of the phase space changes in the sense that chaotic and periodic attractors appear. For both cases we study the chaotic sea by using scaling arguments. We propose an analytical argument to reinforce the validity of the scaling exponents obtained numerically. In principle such an approach can be extended to any two-dimensional map. Finally, based on the Lyapunov exponent, we show that the parameter space exhibits infinite families of self-similar shrimp-shape structures, corresponding to periodic attractors, embedded in a large region corresponding to chaotic attractors.

1. Introduction

With the advance of fast computers, the study of conservative as well as the dissipative nonlinear systems has attracted much attention during the last decades. Introduced in 1969 by Boris Chirikov [1,2], the standard map, or the kicked rotator, becomes one of the most important systems studied in the theory of Hamiltonian systems and area-preserving maps [3,4]. Throughout the years, it has been shown that the standard map can be applied to modelling systems in different fields of science including solid state physics [5], statistical mechanics [6] and accelerator physics [7]. It has also been studied in relation to problems of quantum mechanics and quantum chaos [8–10], plasma physics [11] and many others.

In the absence of dissipation and considering the amplitude of the kicks, namely K, sufficiently small, the structure of the phase space is mixed in the sense that Kolmogorov–Arnold–Moser (KAM) invariant tori and islands are observed coexisting with chaotic seas [12–19] (and references in therein). However, as the parameter K increases and becomes larger than

* Corresponding author. Tel.: +55 19 3526 9174.
E-mail addresses: diegofregolente@gmail.com, diegofregolente@hotmail.com (D.F.M. Oliveira), edleonel@rc.unesp.br (E.D. Leonel).

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$K_c \approx 0.971635\cdots$, the last invariant spanning curve disappears and the system presents a transition from local to global chaos where a chaotic orbit spreads over the phase space leading to an unlimited diffusion of the action [20]. It is important to mention that the standard map can be used only for the regime where the velocities are much smaller than the speed of light, otherwise relativistic corrections must be taken into account.

In this sense we revisit the problem of a particle in the electric field of wave packet. The system is described by a nonlinear two dimensional map. For such a system it has been shown that the energy of the particle cannot grow unlimited due to the existence of invariant spanning curves bordering the chaotic sea [21]. We introduce dissipation into the system considering viscosity and kicking dissipation (dissipation due to inelastic collisions). As we will show and contrary to what is expected, the structure of the phase space is mixed with chaotic and periodic structures ever when the dissipation due to viscosity is introduced. However there is a critical parameter where the invariant curves limiting the size of the chaotic sea are destroyed and a transition from local to global chaos occurs. It means that for some combination of initial conditions and control parameters, the average energy is bounded, but for some other combinations the unlimited energy growth is observed. Such a destruction of the invariant spanning curves does not happen in the pure relativistic standard map [21]. Considering now the case where the dissipation is on the kick, the structure of the phase space is changed and chaotic seas and elliptical fixed points were replaced by chaotic and periodic attractors, each of them with its own basin of attraction. For such a case we have considered two situations, namely (i) strong and (ii) week dissipation. In case (i) we have shown that the system exhibits a period doubling cascade were Feigenbaum's δ was numerically obtained. On the other hand for case (ii) the unlimited energy is suppressed. Based on the Lyapunov exponent, we have show that the parameter space exhibits infinite families of self-similar shrimp-shape structures, corresponding to periodic attractors, embedded in a large region corresponding to chaotic attractors. It is important to emphasise that for the cases of kicking dissipation as well as viscosity, we have studied some statistical properties of the chaotic sea by using scaling arguments. We have found the scaling exponents numerically and their validity were confirmed by a perfect collapse of all the curves of average square action onto an universal plot. Additionally, we have also given some analytical arguments to reinforce the validity of the scaling exponents obtained numerically.

The paper is organised as follows. In Section 2 we construct the two-dimensional map that describes the dynamics of the system. In Section 3 we discuss our numerical results for the conservative dynamics. Section 4 is devoted for the dissipative case. Finally, conclusions are drawn in Section 5.

2. Relativistic particle in a electric field of wave packet

Let us consider the model of a particle in a wave packet [21–26] where the electric field $E(x, t)$ is written as

$$E(x,t) = \sum_{n=-\infty}^{\infty} E_n \sin(k_n x - \omega_n t), \tag{1}$$

where E_n is the amplitude of the nth Fourier component of the electric field wave. Considering that the wave packet has a broad spectrum such that one can assume

$$E_n = E_0, \qquad k_n = k_0 \quad \text{and} \quad \omega_n = n\omega, \tag{2}$$

the electric field $E(x, t)$ can be rewritten as

$$E(x,t) = E_0 \sin(k_0 x) \sum_{n=-\infty}^{\infty} \cos(n\omega t), \tag{3}$$

and therefore by using the Fourier decomposition of the periodic Dirac delta function we obtain

$$E(x,t) = E_0 T \sin(k_0 x) \sum_{n=-\infty}^{\infty} \delta(t - nT), \tag{4}$$

where $T = 2\pi/\omega$. Assuming that the motion of an electron with rest mass m_0 and charge $-e$ in an electric field given by Eq. (4) can be described by the relativistic Hamiltonian

$$H(x,p,t) = \sqrt{p^2c^2 + m_0^2c^4} - \frac{\mathrm{e}E_0T}{k_0}\cos(k_0x) \sum_{n=-\infty}^{\infty} \delta(t - nT), \tag{5}$$

where the speed of light is denoted by c and p is the relativistic momentum given by $p = m_0 v/\sqrt{1-(v/c)^2}$. Thus, from the Hamiltonian above one can obtain the system of the two first order ordinary differential equations for $\dot{x}$ and $\dot{p}$ such as

$$\dot{x} = \frac{\mathrm{d}H}{\mathrm{d}p} = \frac{pc^2}{\sqrt{p^2c^2 + m_0^2c^4}}, \tag{6}$$

$$\dot{p} = -\frac{\mathrm{d}H}{\mathrm{d}x} = -\mathrm{e}E_0T \sin(k_0x) \sum_{n=-\infty}^{\infty} \delta(t - nT) - \frac{\xi pc^2}{\sqrt{p^2c^2 + m_0^2c^4}}. \tag{7}$$