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Network formation by contact arrested propagation

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HIGHLIGHTS

- A new network growth model is presented, termed Contact Arrested Propagation (CAP).
- The model may be formulated on arbitrary networks or on lattices in any dimension.
- We investigate the scaling of model properties and discover universal features.
- Suggested applications of the model include fracture and fragmentation processes.
- The model could be used to generate three-dimensional fracture networks.

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ABSTRACT

We propose here a network growth model which we term Contact Arrested Propagation (CAP). One representation of the CAP model comprises a set of two-dimensional line segments on a lattice, propagating independently at constant speed in both directions until they collide. The generic form of the model extends to arbitrary networks, and, in particular, to three-dimensional lattices, where it may be realised as a set of expanding planes, halted upon intersection. The model is implemented as a simple and completely background independent substitution system.

We restrict attention to one-, two- and three-dimensional background lattices and investigate how CAP networks are influenced by lattice connectivity, spatial dimension, system size and initial conditions. Certain scaling properties exhibit little sensitivity to the particular lattice connectivity but change significantly with lattice dimension, indicating universality. Suggested applications of the model include various fracturing and fragmentation processes, and we expect that CAP may find many other uses, due to its simplicity, generality and ease of implementation.

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1. Introduction

Networks and the processes forming them are topics of interest in a wide range of disciplines. Branching, tree-like networks are common in systems where effective transport of mass, energy or charge is required, including river networks, leaf veins, blood vessels and lightning patterns. Rinaldo, Banavar and Maritan discuss scaling properties of such networks [1] and how they may result from an imperfect search process to optimise network function [2]. Kramer and Marder [3],

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Fig. 1. Snapshots of fracture evolution driven by CO_2 exsolution following fermentation in a confined layer of gelatine containing yeast and sugar. These experiments are presented in Ref. [28].

Takayasu and Inaoka [4] and Leheny and Nagel [5] have shown how river networks may be modelled using simple landscape erosion models, and others have used even simpler models to model drainage network formation, including Leopold and Langbein [6] and Meakin et al. [7], who represented the process by self-avoiding random walks on lattices. Other models for drainage network formation include diffusion limited aggregation models, studied by Meakin [8] and Masek and Turcotte [9], which produce fractal networks.

Fractal networks also arise in percolation systems, which are reviewed by Stauffer and Aharony [10] and by Sahimi [11]. Percolation models have received attention both from mathematicians and from scientists in more applied fields. For example, percolation models have been used for estimating the permeability in petroleum reservoirs [12] or conductivity of disordered materials [13], and invasion percolation models are used to model fluid–fluid displacement processes in porous media [14].

A range of models for generating artificial fracture networks are used in the geosciences to model flow processes in fractured porous media, with applications to, for example, hydrology, petroleum systems or the spread of chemical or nuclear contaminants in geological formations. Adler and Thovert [15] discuss several such network models from a theoretical perspective. Similarly, many authors have proposed simple models of fragmentation processes, including Steacy and Sammis [16], Hernández and Herrmann [17] and Fortes and Andrade [18]. Korsnes et al. [19] devise a fragmentation network model to study the dynamic process of breaking and healing of sea ice, and Iyer et al. [20] use a statistical network approach to model reaction-assisted hierarchical fracturing of rock.

A vast literature has also been devoted to the study of random graphs with random connectivity, starting with Erdös and Rényi [21]. Recently, much attention has been given to the study of so-called complex networks with non-trivial topology, for example social, communication and biological networks. Albert and Barabási review common statistical properties of complex networks and the mechanisms responsible for their formation [22]. They also introduced the preferential attachment model for organisation of complex networks [23]. Watts and Strogatz propose a model that can tune networks from regular to random by starting from a regular graph and rewriting connections randomly [24].

Wolfram [25] has pioneered a variety of network forming processes and models based on simple deterministic rules, and has applied these ideas across scientific disciplines, from the context of biological growth to a fundamental theory of physics. Networks formed by replacing nodes based on the local network structure has been explored further by Morrow et al. [26], Southwell et al. [27] and others.

The model presented here was inspired by the kind of fracture networks described by Kobchenko et al. [28] and illustrated in Fig. 1. Uniform production of CO_2 by a fermentation process causes nucleation of bubbles, and the bubbles develop into fractures, as a consequence of accumulating gas pressure. Fractures propagate until meeting other fractures or an external boundary, allowing the gas to escape through open network pathways. As an idealisation, this process may be viewed as a set of line segments propagating on a lattice according to some rule. The latter idea was refined and generalised into the generic model defined in the next section, which we choose to term Contact Arrested Propagation (CAP).

The CAP model may be related to several of the network forming processes mentioned above. It is general, and may be defined on arbitrary networks, including random graphs, complex networks or lattices (i.e. spatially embedded networks). In the spirit of Wolfram, the model evolves according to a simple deterministic rule, and, when represented on a lattice, it may be studied within the framework of percolation theory. Certain realisations of the CAP model bear resemblance to existing fracture and fragmentation models, such as those referred to above, and this is therefore a field where our model may find useful applications.

In this study, we focus our attention on certain representations of the CAP model on one-, two- and three-dimensional lattices, but investigate them as abstract systems without regard to any particular application. This approach allows us to identify whether aspects of the model's behaviour are generic, or rather artefacts of the representation on a particular background. We return to possible applications of the model in Section 4.2.

2. Model

Before presenting a formal definition of the CAP model, we illustrate it by providing a simple example of a particular model representation. In this example, the elements of our system are the edges of a square lattice, and the edges may

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