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Urban chaos and replacement dynamics in nature and society

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HIGHLIGHTS

- Replacement is a ubiquitous phenomenon in both nature and society.
- Urbanization and urban growth are complex processes of spatial replacement.
- The sigmoid functions can be used to model various types of replacement dynamics.
- Replacement dynamics can be associated with concepts from fractals and chaos.
- A general theory of replacement should be presented to explain complex dynamics.

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ABSTRACT

Replacements resulting from competition are ubiquitous phenomena in both nature and society. The evolution of a self-organized system is always a physical process substituting one type of components for another type of components. A logistic model of replacement dynamics has been proposed in terms of technical innovation and urbanization, but it fails to arouse widespread attention in the academia. This paper is devoted to laying the foundations of general replacement principle by using analogy and induction. The empirical base of this study is urban replacement, including urbanization and urban growth. The sigmoid functions can be employed to model various processes of replacement. Many mathematical methods such as allometric scaling and head/tail breaks can be applied to analyzing the processes and patterns of replacement. Among varied sigmoid functions, the logistic function is the basic and the simplest model of replacement dynamics. A new finding is that replacement can be associated with chaos in a nonlinear system, e.g., urban chaos is just a part of replacement dynamics. The aim of developing replacement theory is at understanding complex interaction and conversion. This theory provides a new way of looking at urbanization, technological innovation and diffusion, Volterra–Lotka's predator–prey interaction, man–land relation, and dynastic changes resulting from peasant uprising, and all that. Especially, the periodic oscillations and chaos of replacement dynamics can be used to explain and predict the catastrophic occurrences in the physical and human systems.

1. Introduction

The components in a self-organized system can always be distributed into two classes, and the process of system's evolution is the process of discarding one kind of component in favor of another kind of component. This process is termed "replacement" [1–4]. For example, the population in a geographical region can be divided into urban population and rural

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population, and urbanization is the process of urban–rural replacement of population; the technology can be divided into new ones and old ones, and technical innovation is the process of new–old replacement of technology. In fact, people can be divided into the rich and the poor, the geographical space can be divided into natural space and human space, and so on. Where there are self-organized systems, there is evolution; and where there is evolution, there is replacement. Replacement results from competition and results in evolution. The replacement phenomena are ubiquitous in both nature and society. Replacement analysis is a good approach to understanding complex systems and complexity.

Logistic function typically reflects a kind of replacement process, and can act as one of the substitution models. In the early literature, the law of logistic replacement was mainly researched in economics, especially in industrial and technological fields. Fisher and Pry [2] once successfully used the logistic function to characterize the substitution of old technology by new. Hermann and Montroll [3] argued that the industrial revolution is a replacement process, that is, the proportion of agricultural workers declines while that of nonagricultural workers increases. Montroll [5] generalized the notion of substitution to a variety of situations by asserting technological and social evolution to be the consequence of a sequence of substitutions of one technique by another. Treating technological innovations as structural fluctuations in a self-organizing industrial system, Batten [6] and Karmeshu et al. [7] gave a conceptual rationale for Fisher–Pry's law of technological replacement. A theoretical progress of the replacement dynamics is the work of Karmeshu [8] and his co-workers, who extended the idea of replacement process for the study of replacement of rural population by urban population and revealed the pattern of urbanization in India. Despite more than 40 years of development, the concept of replacement has not caught the attention of the academic circles. A new progress is that Chen [1,9] and Chen and Xu [10] showed that the replacement dynamics of urbanization and urban growth can be associated with complex dynamics such as bifurcation and chaos.

Replacement phenomena can be found everywhere, and the logistic model is not sufficient to describe a variety of substitution behavior. All the sigmoid functions can be employed to model replacement dynamics, but the logistic function seems to be the simplest and basic one, and thus the most important one. Besides the mathematical models, we need other measurements and quantitative tools to analyze replacement processes. Today, it is time to develop a general theory of replacement dynamics by means of the idea from fractals, chaos and urban replacement. The new points of this work lie in three aspects. First, a general principle of replacement dynamics is proposed. Second, the two-group interaction model is employed to interpret the process of replacement. Third, the periodic oscillations and chaos of replacement dynamics are used to explain the catastrophic occurrences in natural and social evolution. The base of this study is urbanization and urban growth. The rest of this paper is organized as follows. In Section 2, the theory of urbanization dynamics is advanced, and the 1-dimensional logistic map is linked with the 2-dimensional map of the two-population interaction. In Section 3, the model of replacement dynamics is generalized to different fields such as ecology, geography, and history. In Section 4, a general theory of replacement dynamics is preliminarily propounded, and the related questions are discussed. Finally, the paper is concluded by summarizing the mains of this work.

2. Models

2.1. The 1-D logistic map

Cities and systems of cities are self-organized systems [11,12], and urbanization is a process of phase transition that is associated with replacement. The *level of urbanization* is the basic measurement describing the extent of regional urbanization. It is always defined with the ratio of the urban population to total population in a geographic region, that is

$$L(t) = \frac{u(t)}{P(t)} \times 100\% = \frac{u(t)}{r(t) + u(t)} \times 100\%, \tag{1}$$

where $u(t)$ refers to the urban population of time t, $r(t)$ to the rural population at the same time, $P(t) = u(t) + r(t)$ is the total population, and $L(t)$, the level of urbanization of a region. The measurement equivalent to the level of urbanization is the *urban–rural ratio* (URR), which is defined by $o(t) = u(t)/r(t)$ [10,13]. By means of these simple measurements, the relation between the 1-dimensional logistic map and a 2-dimensional two-population interaction map can be brought to light [9].

In urban studies, one of the basic problems concerning geographers is how the level of urbanization changes over time. By mathematical knowledge, if some kind of measurement of a system has clear upper limit and lower limit, the growing course of the system always takes on an S-shaped curve. The curve can be formulated as a *sigmoid function*. The sigmoid function is also called *squashing function*. Pressed by the upper limit and supported by the lower limit, a line will be twisted into S shape. The family of sigmoid functions includes various functions such as the ordinary arc-tangent, the hyperbolic tangent, and the generalized logistic function. Among all these sigmoid functions, the simplest and the well-known one is the logistic function. Sometimes, the logistic function is the synonym of the sigmoid function [14].

As we know, the level of urbanization has clear upper and lower limits and comes between 0 (or 0%) and 1 (or 100%), thus it can often be fitted to the logistic function [4,8–10]. For many years, the United Nations experts of urbanization employed the logistic model to predict the level of urbanization of different countries [13,15]. The logistic model of urbanization can