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Electron Bernstein waves in nonextensive statistics

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HIGHLIGHTS

- The dispersion relation of Bernstein waves in nonextensive plasmas is derived.
- The nonextensive q-distribution significantly modifies the Bernstein dispersion relation.
- Bernstein waves are studied in both the strongly and weakly magnetized regimes.
- The dispersion solutions of Bernstein waves are derived for asymptotic cases.

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ABSTRACT

The propagation of electron Bernstein waves in a magnetized plasma is investigated in the context of nonextensive q-distribution of Tsallis statistics. The dispersion relation is expressed as a function of generalized hypergeometric function. It is shown that the generalized dispersion relation significantly depends on the q-parameter, which quantifies the degree of nonextensivity of the system. The reduced number of superthermal particles shifts the Bernstein wave curves to higher wavenumbers. For harmonics whose frequency lies above the upper hybrid frequency, an increase in the value of q increases the maximum frequency and the wavenumber at which the group velocity vanishes. For Bernstein waves which propagate at frequencies lower than the upper hybrid frequency or close to it, diminishing q, or increasing the number of superthermal particles, gives rise to faster frequency fall-off. The generalized electron Bernstein waves are studied in both the strongly and weakly magnetized regimes and for both small and large wavenumbers compared with the Larmor radius. It is found that for the weak magnetic field, the frequency range occupied by the mode spans the complete intraharmonic frequency, and the q value significantly affects the dispersion curves. On the other hand, in strongly magnetized regimes the frequency variation domain is extremely restricted and has little dependence on the q value.

1. Introduction

Bernstein waves are special electrostatic waves perpendicularly propagating in a magnetized plasma and always localized near the cyclotron harmonics. These waves which were first derived by Bernstein [1] can propagate undamped very close to the plane perpendicular to the magnetic field. The first experimental observation of the electron Bernstein waves was reported by Crawford [2] and by Leuterer [3] and of ion Bernstein waves was reported by Schmitt [4]. Bernstein waves have also been frequently observed by spacecrafts, emitted from the terrestrial magnetosphere [5], and from the

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magnetospheres of other planets such as Jupiter [6], Saturn [7], Uranus [8], and from the magnetized plasma of Jupiter's moon Io [9–11]. Such waves are a preferred source of plasma heating because they strongly interact with electrons [12]. Electron Bernstein waves also have been used for inferring the local electron temperature [11,13] by fitting the measurements to the wave dispersion.

In the kinetic theory the distribution functions are used to describe the properties of the Bernstein waves. In Boltzmann–Gibbs (B–G) statistics the Maxwellian distribution is believed valid universally for the macroscopic equilibrium systems. However, many space and laboratory plasmas are known to possess particles whose velocity distribution functions are not Maxwellian. For example, planetary magnetospheres [6–8], astrophysical plasmas, solar wind [14,15] and tokamak edge plasma [16] show non-Maxwellian behavior.

In recent years, a family of so-called kappa velocity distributions, or generalized Lorentzian distribution, introduced first by Vasyliunas [17] is recognized to be appropriated for the investigation of Bernstein waves in nonequilibrium plasma systems: for instance, Mace [18,19] used a Gordeyev integral for the investigation of electron Bernstein modes in a plasma with an isotropic kappa velocity distribution; a numerical investigation of the dispersion relation for ion Bernstein waves in a kappa distributed plasma has been carried out by Nsengiyumva et al. [20]; Deeba et al. [21] derived Q3 the generalized dielectric constant for the electron Bernstein waves for both anisotropic kappa and the generalized (r, q) distributions.

Q4 In the following years, a great deal of interest has been devoted to a new statistical approach based on the generalization of the B–G entropy, first recognized by Renyi [22] and subsequently suggested by Tsallis [23]. It is described by a nonextensive Q5 parameter q which specifies the degree of nonextensivity. For $q \neq 1$ it gives power-law distribution functions, and only when the parameter $q \to 1$ Maxwellian distribution is recovered [23]. This nonextensive q-distribution function has been employed successfully in a wide range of systems characterized by nonextensivity such as plasma systems [15,24–40]. Note that because of a lack of formal derivation, Leubner [41] suggested a nonextensive approach to kappa distributions. Rios et al. [42,43] obtained a reduced kappa distribution, which can be transformed into a nonextensive q-distribution by the expression $\kappa = 1/(q-1)$. The standard kappa distribution is known close to the nonextensive q-distribution; however, there is no simple transformation between the two distributions as their arguments and powers do not fit the same transformation [27]. The aim of the present work is therefore to investigate and illustrate the propagation characteristics of electron Bernstein waves for nonextensive q-distributions. We study the behavior of the dispersion relation for the weakly and strongly magnetized nonextensive plasmas.

2. Theoretical model

We start with some basic facts about the nonextensive q-distribution function in Tsallis statistics. In the nonextensive framework q is a parameter quantifying the degree of nonextensivity of the system and the distribution function is given by Ref. [44]

$$f_\sigma(\mathbf{v}) = f_\sigma(v_x, v_y, v_z) = n_{\sigma 0} B_q \left[1 - (q-1)\frac{v^2}{v_{T\sigma}^2}\right]^{\frac{1}{q-1}} \quad (1)$$

where the normalization constant reads

$$B_q = \frac{\sqrt{1-q}}{\pi^{3/2} v_{T\sigma}^3}\left(\frac{3q-1}{2}\right)\frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q}-\frac{1}{2}\right)}, \quad \text{for } -1 < q \le 1 \quad (2)$$

and

$$B_q = \frac{\sqrt{q-1}}{\pi^{3/2} v_{T\sigma}^3}\left(\frac{3q-1}{2}\right)\left(\frac{q+1}{2}\right)\frac{\Gamma\left(\frac{1}{q-1}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)}, \quad \text{for } q \ge 1 \quad (3)$$

where $\sigma(=e, i)$ denotes electron (e) or ion (i), $n_{\sigma 0}$ is the particle number density, Γ is the gamma function, $v_{T\sigma} = \sqrt{2k_B T_\sigma / m_\sigma}$ is the thermal velocity, and T_σ and m_σ are the temperature and mass for the species σ, respectively. It is important to mention that the temperature T_σ is the kinetic temperature which the species would have in the absence of nonextensive effects $(q \to 1)$. It is possible to derive the effective q-temperature, $T_{\text{eff}}(q) = \frac{2T_\sigma}{5q-3}$, by applying equipartition of energy and defining $\frac{3}{2}k_B T_{\text{eff}} \equiv \langle \frac{1}{2}mv^2 \rangle$. It is worth noting that there is a temperature dependent cut-off on the magnitude of the velocities for $q > 1$, which is given by $v_{\max} = v_{T\sigma}/\sqrt{(q-1)}$. Further, using that $\lim_{|z|\to\infty}[\Gamma(a+z)/\Gamma(z)]z^{-a} = 1$ [45], it is easy to see that the Maxwellian distribution is recovered in the limit $q \to 1$.

Liu et al. have shown [32] that in the case of the nonextensive q-distributions with $q < 1$, compared with the Maxwellian distribution, there are more superthermal particles, i.e. particles with the speed faster than the thermal speed, when the