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Permutation approach, high frequency trading and variety of micro patterns in financial time series

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HIGHLIGHTS

- We investigate high frequency financial time series in micro scale.
- We introduce a method for the investigation of micro patterns.
- Detection of dynamics and comparing degree of complexity of financial time series.
- HFT in recent years affect the micro patterns which may be seen in financial time series.

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ABSTRACT

Permutation approach is suggested as a method to investigate financial time series in micro scales. The method is used to see how high frequency trading in recent years has affected the micro patterns which may be seen in financial time series. Tick to tick exchange rates are considered as examples. It is seen that variety of patterns evolve through time; and that the scale over which the target markets have no dominant patterns, have decreased steadily over time with the emergence of higher frequency trading.

1. Introduction

In the middle of sixties, the algorithmic complexity theory was independently developed by Kolmogorov [1] and Chaitin [2]. To parameterize complexity in deterministic or random dynamical systems, the most important quantity which may be used is entropy. There are different ways to count the diversity of any pattern generated by a data source: Shannon entropy, metric entropy, topological entropy, etc. After the seminal works of Shannon [3], in 1949 the word entropy came to the fore in the new context of information theory, coding theory, and cryptography. Recently the concept of entropy is also used in econophysics (see Refs. [4,5] and references therein) and sociodynamics [6]. The concept of entropy has been evolved along different ways: Renyi entropy [7], topological entropy [8], Tsallis entropy [9], directional entropy [10], permutation entropy [11], epsilon–tau entropy [12], etc. Permutation entropy was introduced by Bandt, Keller, and Pompe in Refs. [13,14]. Entropies are basic observables for dynamical systems. In Ref. [14] a piecewise monotone map f from an interval I into itself is defined, and it is shown that for piecewise monotone interval maps the Kolmogorov–Sinai entropy can be obtained from

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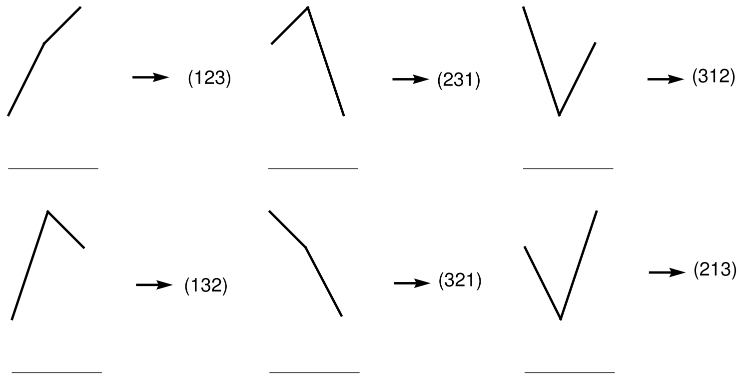

Fig. 1. Permutations $\Pi_i,\ i = 1, 2, \dots, 6$, for $n = 3$.

order statistics of the values in a generic orbit. It has been shown that it is possible to use the permutation entropy to detect dynamical changes in a complex time series [15].

Recently, permutation entropy has been used to study dynamical changes of EEG data [16]; and based on permutation entropy, mutual information of two oscillators has been calculated [17].

In this article variety of micro patterns in financial time series have been studied. Variety of patterns in financial time series is an important measure. In a completely random series all different patterns may occur with equal weight. If some patterns in a time series are much less than the others, the time series contains dominant patterns. These dominant patterns represent some characteristics of the system, which needs to be revealed. This information also could be used for prediction of future changes, which for financial time series represent inefficiency in the target market. There are several definition for the efficient market hypothesis (EMH) [18–20]. According to the EMH hypothesis, asset prices move as random walks over time, and technical analysis should provide no useful information to predict future changes [21]. This means that asset prices in an efficient market fluctuate randomly in response to the unanticipated component of news [22]. Some people take EMH as a core assumption in finance theory [23]. But many physicists consider it only an approximation [24]. According to EMH, in an efficient market variety of patterns for increments of the price in different scales should be at maximum level.

It is now known that most markets behave efficiently in macro scale, and there are no dominate patterns in their financial time series. But it seems that the investigation of micro patterns, and searching for dominant patterns by participants in markets, in recent years, have faded those patterns. Permutation entropy is taken as a criteria for measuring the variety of micro patterns which may be seen in financial time series. Tick to tick exchange rate time series are considered as an example.

2. Definition

Consider a set of n distinct real numbers, $\{a_1, a_2, \dots, a_n\}$. One may define a map from these numbers to the set $\{1, 2, \dots, n\}$ in such a way that the ordering of the second set is the same as the first one. The range of this map will be $n!$ permutations. The permutation corresponding to $\{a_1, a_2, \dots, a_n\}$ is called the *pattern*, and is denoted by Π. See Fig. 1, for the case $n = 3$. Consider a time series $\{x_i\}_{i=1,\dots,N}$. By a window of length n, we mean any subsequence of the form $\{a_m, a_{m+1}, \dots, a_{m+n}\}$. There are $N - n + 1$ windows of length n, to each of them there corresponds a pattern Π. If Π_i is a given pattern, we define

$$p_i := \frac{N_i}{N - n + 1}, \tag{1}$$

where N_i is the number of n consecutive numbers with pattern Π_i. For large N, p_i tends to the probability of occurring the pattern Π_i. Permutation entropy of order n of a time series, $\{x_k\}_{k=1}^{N}$, is defined as (see e.g. Ref. [13])

$$H_n := -\sum_{i=1}^{n!} p_i \log p_i. \tag{2}$$

It can be shown that $0 \le H_n \le \log n!$ [25]. Upper bound occurs when all p_i's have the same value, *i.e.* when the time series is a completely random series; and the lower bound occurs when only one of p_i's is nonzero, which happens when the time series is a decreasing or increasing sequence.

In some cases a linear function $H_n = k(n-1) + C$ is a good approximation. This means that for these time series, in the view of permutation entropy, there are just two degrees of freedom for the time series.[1]

[1] Of course this is not a common case, for example, in a completely random time series $H_n \propto n \ \ln n$, for large n, which is not a linear function of n.