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Statistical properties of position-dependent ball-passing networks in football games

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HIGHLIGHTS

- We propose a method for creation of a position-dependent ball passing networks.
- The networks possess the small-world property.
- The degree distribution of each network is fitted well by the truncated gamma distribution function.
- Statistical properties of the networks are reproduced by the numerical model based on a Markov chain.
- Our method and model offer a unified view for the results of previous studies.

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ABSTRACT

Statistical properties of position-dependent ball-passing networks in real football games are examined. We find that the networks have the small-world property, and their degree distributions are fitted well by a truncated gamma distribution function. In order to reproduce these properties of networks, a model based on a Markov chain is proposed.

1. Introduction

Scientific studies on sports activities have been carried out in a wide variety of research fields such as psychology, physiology, biomechanics, and also physics. Among various sports events, football is one of the major subjects [1]. From the viewpoint of physics, studies with respect to football games are considered to be classified into the following two types: mechanical and statistical. In the latter case, goal distribution [2–5] and outcome prediction of football games [6–9] have been studied for example. A football game can be considered as a dynamical system in which game players interact with each other via one ball. Ball-passing events have been focused on in various statistical analyses for the collective behavior of players [10,11], temporal sequences of players' action [12] and ball movements [13], and passing sequence to goal [14].

In statistical physics, analysis for complex networks has achieved rapid development recently [15,16]. The network analysis has already been applied to football games such as a structural property of ball-passing networks [17,18], and assessment of players [19,20]. In the studies of the ball-passing networks [17,18], each node and edge of the network represent an individual player and passing of the ball, respectively. One main conclusion in their studies is that ball-passing networks of football games have the scale-free property, namely the degree distributions follow the power law. However, in

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Nomenclature

N	The total number of nodes
M	The total number of edges
ℓ	Mean path length
C	Clustering coefficient
k	Degree
$f(k)$	Probability distribution of degree
$F(k)$	Cumulative distribution of degree
ν	Shape parameter of the truncated gamma distribution
λ	Scale parameter of the truncated gamma distribution
$a_i^{(t)}$	Ball-possession probability for the node i at time t
$\boldsymbol{a}^{(t)}$	Probability vector of $a_i^{(t)}$
$g(a)$	Probability distribution of $\boldsymbol{a}^{(500)}$
$G(a)$	Cumulative distribution of $\boldsymbol{a}^{(500)}$
$P_{i\to j}$	Ball-passing probability from the node i to the node j
$\boldsymbol{P}$	Transitive matrix
r_{ij}	The distance between the two nodes i and j
L_j	The distance of the node j from its home position
$Q_\alpha(r_{ij})$	The factor for the distance of passes in $P_{i\to j}$
$R_{\beta,\xi}(L_j)$	The factor for the existence probability of the player receiving a pass
α	The parameter in $Q_\alpha(r_{ij})$
β	The parameter in $R_{\beta,\xi}(L_j)$
ξ	The parameter in $R_{\beta,\xi}(L_j)$
L_{relax}	The characteristic moving distance of each player
g	The ball-passing probability to the opponent team

their network analysis, the total number of nodes were only 11, which was the number of players on a ground in one team. Clearly, it is too few to judge the power-law behavior of the degree distributions.

In this paper we propose another method for creating a ball-passing network and report statistical properties of the network. Since each player has his own role corresponding to their home position and it is an important factor for ball passing, we create a *position-dependent* network in the next section. In Section 3, the structural properties and the degree distributions of the networks obtained from real games are examined. We find that the degree distributions can be fitted with a truncated gamma distribution in common. In Section 4, we propose a numerical model based on a Markov chain by introducing the ball-possession probability. Discussion and conclusion are given in Sections 5 and 6, respectively.

2. Method for creating a ball-passing network

A "position-dependent" network of ball passing, where the position of a player in a soccer field is considered, was obtained by the following method. In order to specify the position of a player, we divide the field into 18 areas (six areas along the goal direction, and three areas along the vertical direction in Fig. 1). Note that, this is the same division as used in the FIFA official statistical data of 2010 World Cup South Africa [21] and in the Ref. [12]. An area expressed in the coordinate (x, y) is labeled by the area number $A_{xy} = 6(y-1) + x$ $(1 \le x \le 6,\ 1 \le y \le 3)$. A node is assigned to a player on one of the 18 areas, and labeled by the node number $11(A_{xy} - 1) + u$ $(1 \le u \le 11)$. The total number N of nodes for one team is $198(=18 \times 11)$.

When one player passes the ball to another player in the game, two nodes corresponding to these two players are connected by an undirected edge. If more than one passes are made between the same nodes, multiple edges are allowed. A ball-passing network is obtained as the set of the all passes made by one team in a game. To be precise, we use the following rules: (i) only the passes between players belonging to the same team are considered; (ii) when a player is replaced by a reserved player, the node for the new player is given the same number as the old player. From Fig. 1, for example, we obtain the network among the three nodes "159$(u = 5)$", "106$(u = 7)$", and "131$(u = 10)$" as shown in Fig. 2.

We focus on the following basic properties characterizing the network structure: the total number M of edges, the degree distribution $f(k)$, the average degree $\langle k\rangle$, mean path length ℓ, and clustering coefficient C. The mean path length l is defined as the total average of $d(i, j)$, which is the network distance from i to j. The clustering coefficient C is defined as the total average of C_i, which is the clustering coefficient for node i given as

$$C_i = \frac{T_i}{k_i(k_i - 1)/2}. \tag{1}$$

Here k_i denotes the degree of node i, and T_i denotes the number of the triangles containing the node i.