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Robustness of interdependent and interconnected clustered networks

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HIGHLIGHTS

- As coupling strength increases, the system changes from second order through hybrid order to first order phase transition.
- For weak coupling strength, corresponding to second order transition, clustering has almost no effect on the robustness of network, but for strong coupling strength, corresponding to first order transition, the system that is more clustered is more vulnerable.
- When the system is more clustered, the hybrid order region is almost unchangeable, the first order region becomes smaller, and the second order region is larger.

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ABSTRACT

In real world, most systems show significant clustering, and it is more practical to investigate the behaviors of clustered network. Previous studies are mostly focused on single clustered network and coupled clustered networks with dependency links. Here we study two clustered networks coupled with both interdependent and interconnected links by introducing generating function of the joint degree distribution. When the networks are fully dependent, we obtain the analytical solution of giant component P_∞. We show rich phase transition phenomena and analyze their behaviors. We find that, as dependency coupling strength increases, the system changes from second order phase transition through hybrid transition to first order phase transition. For weak dependency coupling strength q_A, corresponding to second order phase transition, we find that, clustering has almost no effect on the robustness of network, but for strong dependency coupling strength q_A, corresponding to first order transition, the more clustered system is more vulnerable. At the same time, we notice that when the system is more clustered, the hybrid order region is almost unchangeable, the first order region becomes smaller, and the second order region is larger. Additionally, we can see that, the bigger the clustering coefficient c is, the bigger the second order region becomes. For the same c, the density of connectivity links between networks is higher, the second order region becomes smaller, and the density of connectivity links within each network is higher, the second order region becomes bigger.

1. Introduction

During the last decade, the properties of complex networks have been studied intensively [1–17]. And most real networks are not isolated, they interact with each other [18–23]. Besides, many networks, e.g., the Internet, scientific collaboration

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networks, metabolic and protein networks and movie actors networks, show significant clustering [24–29]. Recent studies demonstrated many long-standing problems of complex networks, such as how clustering affects robustness of networks and how to construct a plausible and resilient network with clustering in single network [30–38].

Newman [37] proposed random-graph model of a clustered network and solved for many of its properties by extending the method of generating function. Adam et al. [39] provided a general analytical expression for the expected size of a cascade outbreak and considered how clustering qualitatively affected the cascade condition. They emphasized that for site and bond percolation, clustering will unambiguously decrease the cascade size. Based on the model of highly-clustered networks, Gleeson et al. [40] found that clustering increased the bond percolation threshold in comparison with its value for non-clustered networks with the same degree distribution and correlation structure. These findings highlight the need to consider the percolation properties of single clustered network for designing robust networks.

However, in our modern life, most networks are not isolated, they are coupled with each other by interdependent links or interconnected links or both. Recently, Buldyrev et al. [41] studied the robustness of two interacting unclustered networks, which only exist as dependency links. They found that the interdependence caused the system to be significantly less robust [41,42]. At the same time, Leicht and D'Souza [43] investigated the interacting unclustered networks coupled only by connectivity links. They derived that the interconnected links made the system more robust. More recently, Huang et al. [44] studied the robustness of two interacting clustered networks coupled with dependency links. They found that the two clustered interdependent networks became more vulnerable in contrast to single clustered network. Shao et al. [45] studied the robustness of n clustered networks coupled with partially dependency links. Their results showed that the influence on robustness of network due to clustering coefficient became smaller as decreasing the dependency coupling strength. Hu et al. [46] studied the percolation in a system of two random unclustered networks, which contains interdependent and interconnected links. They found that changing the strength of interconnecting links can change the transition from second order phase transition through hybrid phase transition to first order.

About interacting networks, previous studies are mostly focused on random clustered networks with dependency links and random unclustered networks with dependency and connectivity links. However, clustered networks with both dependency and connectivity links are more complicated and realistic, understanding the robustness is one of the major challenges for designing resilient infrastructures.

2. The model

In this paper, we consider a system of two interdependent and interconnected clustered networks A and B with the number of nodes N_A and N_B, respectively. On one hand, a node from one network depends on no more than one node from the other network with non-feedback condition. We assume that, a fraction q_A of network A nodes depends on nodes in network B and a fraction q_B of network B nodes depends on nodes in network A. On the other hand, connectivity links are established by randomly connecting the nodes within each network and between the two networks according to the joint degree distributions $P^A_{s_A,t_A,k_{AB}}$ and $P^B_{s_B,t_B,k_{BA}}$, where $P^A_{s_A,t_A,k_{AB}}$ ($P^B_{s_B,t_B,k_{BA}}$) represents the probability of a node within network A (B) to have s_A (s_B) single edges and t_A (t_B) triangles to other A (B) nodes and k_{AB} (k_{BA}) links towards B (A) nodes. The conventional degree of each node k_A (k_B) is equal to $s_A + 2t_A$ ($s_B + 2t_B$). When the density of triangles of network is approximative to zero, it is corresponding to the situation of random network.

Given the conventional joint degree distribution of network A, the probability $P^A_{k_A,k_{AB}}$ that a node in network A has k_A links to other A nodes in total and k_{AB} links towards B nodes, is $P^A_{k_A,k_{AB}} = \sum_{s_A,t_A=0}^{\infty} P^A_{s_A,t_A,k_{AB}} \delta_{k_A,s_A+2t_A}$, where δ_{k_A,s_A+2t_A} is the Kronecker delta. Then, we can write down a generating function for the conventional joint degree distribution $P^A_{k_A,k_{AB}}$ [43,44,46]:

$$F_0^A(z_A, z_B) = \sum_{k_A,k_{AB}=0}^{\infty} P^A_{k_A,k_{AB}} z_A^{k_A} z_B^{k_{AB}} = \sum_{k_A=0}^{\infty} \sum_{k_{AB}=0}^{\infty} \sum_{s_A,t_A=0}^{\infty} P^A_{s_A,t_A,k_{AB}} \delta_{k_A,s_A+2t_A} z_A^{k_A} z_B^{k_{AB}}$$
$$= \sum_{k_{AB}=0}^{\infty} \sum_{s_A,t_A=0}^{\infty} P^A_{s_A,t_A,k_{AB}} z_A^{s_A+2t_A} z_B^{k_{AB}} = G_0^A(z_A, z_A^2, z_B).$$

In this manner, we define three dimensional generating functions of the joint degree distributions describing all the connectivity links according to Ref. [37],

$$G_0^A(x_A, y_A, z_B) = \sum_{s_A,t_A,k_{AB}=0}^{\infty} P^A_{s_A,t_A,k_{AB}} x_A^{s_A} y_A^{t_A} z_B^{k_{AB}}, \qquad G_0^B(x_B, y_B, z_A) = \sum_{s_B,t_B,k_{BA}=0}^{\infty} P^B_{s_B,t_B,k_{BA}} z_A^{k_{BA}} x_B^{s_B} y_B^{t_B}.$$

And the clustering coefficient of network A is

$$c_A = \frac{3 \times (\text{number of triangles in network})}{\text{number of connected triples}} = \frac{N_A \sum_{s_A,t_A} t_A P^A_{s_A,t_A}}{N_A \sum_{k_A} \binom{k_A}{2} P^A_{k_A}},$$