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Probabilistic solution of vibro-impact stochastic Duffing systems with a unilateral non-zero offset barrier

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HIGHLIGHTS

- The stationary PDF of vibro-impact stochastic Duffing systems is studied.
- A unilateral non-zero offset barrier in the vibro-impact system is considered.
- A proposed solution procedure consists of three steps for the PDF solution.
- Four different cases are studied to show the effectiveness of the solution procedure.
- The tail region of the PDF is well approximated by the solution procedure.

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ABSTRACT

This paper proposes a solution procedure for obtaining an approximate probability density function (PDF) of the response of vibro-impact Duffing systems with a unilateral non-zero offset barrier. The impact model is used with classical impacts with instantaneous velocity jumps and the excitation is modelled by a zero mean stationary Gaussian white noise. First the original vibro-impact system is converted into a system without barriers by adopting the Zhuravlev non-smooth coordinate transformation. Second the PDF of the converted system is obtained by solving the Fokker–Planck (FP) equation with the exponential–polynomial closure (EPC) method. Last the PDF of the original system is achieved using the well-established methodology on seeking the PDF distribution of a function of a random variable. A further parametric study is also conducted to evaluate the effectiveness of the proposed solution procedure. Comparison with the simulated result shows that when lightly inelastic impacts occur (i.e, the restitution coefficient is very close to 1.0), the technique of the Zhuravlev non-smooth coordinate transformation is feasible. The transformed system can adequately represent the original system. Consequently the EPC method can provide a good approximate PDF solution. The tail region of the PDF is also approximated well. The analysis further shows that the non-zero offset significantly affects the shape of the PDF distribution of displacement.

1. Introduction

Vibro-impact systems widely exist in the field of engineering, such as structural interaction in piping systems [1], nonlinear soil–structure interaction [2], heat exchanger tubes under aerodynamic excitation, vibratory pile drivers, gear-pair systems with backlash, and ship's motion when colliding against fenders [3]. In order to model actual impacts, there are some

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modelling techniques developed to model impacts, such as Hertz contact law, classical impacts with instantaneous velocity jumps, and power-law phenomenological modelling. There have been lots of relevant researches done using different impact models because excessive or severe impacts may cause the damage of some structural components or the overall failure of systems in practice. A simple way for avoiding harmful impacts is to enlarge gaps. However, simply increasing gaps may not effectively fulfil the safety target in some cases, e.g., the cases of some systems experiencing random vibration. Therefore, the investigation on the response of vibro-impact systems in different situations has received much attention from the scientific community in the past few decades. The response of vibro-impact systems under deterministic excitation has been widely studied on some interesting problems, e.g., subharmonic vibration [4], impact mode formulation [5], chaotic motion [6–8] and grazing bifurcation [9–11]. Comparatively, the behaviour of vibro-impact systems under random excitation is also of significance to be studied. In this research field, the stochastic response of vibro-impact systems is evaluated by statistical moments or probability density functions (PDFs) [12,13].

As the PDF solution is considered, it is well known that the number of available exact PDF solutions to the Fokker–Planck (FP) equation is quite limited [14]. Most problems have to rely on approximate approaches or numerical methods. For exact solutions on vibro-impact problems, only a few exact stationary PDF solutions can be obtained for the case of perfectly elastic, instantaneous impacts, which are subject to some special requirements [15]. Jing and Sheu developed some closed-form solutions for the stationary response of a single-degree-of-freedom vibro-impact system with the Hertz contact law under Gaussian white noise by solving the FP equation [16]. After that, Jing and Young extended the solution technique to solve the problems of a vibro-impact system with clearance and impact interactions between two vibration systems under random excitation [2,17]. For approximate PDF solutions, quite a few studies developed stochastic averaging methods to obtain the PDF solutions. Huang et al. applied a stochastic averaging method to deal with the problem of multi-degree-of-freedom vibro-impact systems with the Hertz contact law [18]. Sri Namachchivaya and Park used a stochastic averaging method to investigate the vibro-impact oscillator with two-sided barriers [19]. In their study, the model of classical impacts with instantaneous velocity jumps was used. In this model, a restitution coefficient is applied between impact velocity and rebound velocity according to Newton's law. The restitution coefficient is used to evaluate the degree of impact losses. Using the impact model with a restitution coefficient, Feng et al. adopted a stochastic averaging method to study the response of vibro-impact Duffing oscillators and vibro-impact Duffing–Van der Pol oscillators with a zero-offset barrier [20,21]. In their studies, the Zhuravlev non-smooth coordinate transformation is adopted to convert a vibro-impact oscillator into an oscillator without barriers [22]. The stationary PDF of the converted oscillator is governed by the FP equation which is solved by a stochastic averaging method. More recently, the stochastic averaging method with the Zhuravlev non-smooth coordinate transformation technique was extended to investigate the response probability density functions of a Duffing–Van der Pol vibro-impact system under correlated Gaussian white noise excitations [23]. Besides, Monte Carlo simulation is another versatile technique to obtain the PDF solution of vibro-impact oscillators in a straightforward manner [24]. The constraints of either a unilateral barrier or two-sided barriers can be considered in the simulation. However, extensive computational efforts are spent if the tail region of the PDF solution is studied. The process of impact tracing also produces many additional points around the barrier when catching each impact. In order to simulate an adequate PDF, both response interval selection and sample selection around the barrier are crucial in this simulation technique.

As above described, the random vibration of vibro-impact systems has received much research interest in the past few decades. However, the investigation on the PDF solution of vibro-impact systems mostly relies on stochastic averaging methods. The stochastic averaging method is suitable in the case of lightly damping and weak wide-band excitation. Furthermore, the tail PDF of the response is less developed in previous studies. These limitations motivate this study on the PDF solution of vibro-impact systems under random excitation. This paper is devoted to developing a solution procedure to seek the approximate PDF solution of vibro-impact systems with a unilateral non-zero offset barrier. The model of classical impacts with instantaneous velocity jumps is used in this study. In the first step, the Zhuravlev non-smooth coordinate transformation is applied to the equation of motion such that the transformed equation does not contain any impact terms. In the second step, the PDF of the response of the transformed system is governed by the FP equation which is solved by the exponential–polynomial closure (EPC) method [25–29]. In the last step, the PDF of the original system is achieved using the well-established methodology on seeking the PDF distribution of a function of a random variable [30]. A parametric study is further conducted on a vibro-impact Duffing system with a unilateral non-zero offset barrier to show the effectiveness of the proposed solution procedure. Comparison with the simulated result shows that the proposed solution procedure can present a good approximate PDF solution for the examined system, especially for the tail region. The analysis further shows the shape of the displacement PDF is significantly affected by the non-zero offset.

2. Problem formulation

A vibro-impact Duffing system under a unilateral non-zero offset barrier can be formulated by

$$\ddot{y} + c\dot{y} + ky + \mu y^3 = \xi(t), \quad y > b, \tag{1}$$

$$\dot{y}_+ = -r\dot{y}_-, \quad y = b,\ 0 < r \le 1. \tag{2}$$

Eqs. (1) and (2) express the equation of motion for the vibro-impact systems as shown in Fig. 1. Herein $\ddot{y},\ \dot{y},\ y$ represent the acceleration, velocity and displacement of the mass (mass m is simply taken as unity in Fig. 1), respectively; c denotes