Physica A 413 (2014) 77–85

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The use of copula functions for modeling the risk of investment in shares traded on the Warsaw Stock Exchange

Krzysztof Domino *, Tomasz Błachowicz

Institute of Physics - Centre for Science and Education, Silesian University of Technology, Krzywoustego str. 2, 44-100 Gliwice, Poland

HIGHLIGHTS

- Shares prices of companies traded on the Warsaw Stock Exchange were investigated.
- Copula functions were used to model the risk of the investment in shares.
- The Hurst exponent was calculated using the local Detrended Fluctuation Analysis.
- The Hurst exponent became a useful tool in the copula selection procedure.

ARTICLE INFO

Article history:
Received 7 February 2014
Received in revised form 9 May 2014
Available online 5 July 2014

Keywords:
Econophysics
Warsaw Stock Exchange
Copula functions
Hurst exponent
Detrended fluctuation analysis
Copula selection procedure

ABSTRACT

In our work copula functions and the Hurst exponent calculated using the local Detrended Fluctuation Analysis (DFA) were used to investigate the risk of investment made in shares traded on the Warsaw Stock Exchange. The combination of copula functions and the Hurst exponent calculated using local DFA is a new approach. For copula function analysis bivariate variables composed of shares prices of the PEKAO bank (a big bank with high capitalization) and other banks (PKOBP, BZ WBK, MBANK and HANDLOWY in decreasing capitalization order) and companies from other branches (KGHM—mining industry, PKNORLEN—petrol industry as well as ASSECO—software industry) were used. Hurst exponents were calculated for daily shares prices and used to predict high drops of those prices. It appeared to be a valuable indicator in the copula selection procedure, since Hurst exponent's low values were pointing on heavily tailed copulas e.g. the Clayton one.

1. Introduction

The evaluation of the Value at Risk (VaR) type model [1,2] of risk management and risk determination has been a challenge in recent 25 years. Recently copula functions have been used to analyze financial data and evaluate VaR type models [3]. In this paper we combined the VaR model and copula approach with the concept of Anomalous Diffusion (AD), Detrended Fluctuation Analysis (DFA) and the Hurst exponent [4–14]. This is a new approach and, as we believe, it will contribute to financial data analysis in analogy to physical complex dynamical system analysis [15]. The model discussed here was used to analyze financial data i.e. daily shares prices of companies traded on the Warsaw Stock Exchange. Shares prices of the PEKAO bank (a big bank with high capitalization) were composed with shares prices of other banks (PKOBP, BZ WBK, MBANK and HANDLOWY in decreasing WIG20 participation order) to create bivariate variables. Similarly shares prices of PEKAO were combined with shares prices of companies from other branches (mining industry—KGHM, petrol industry—PKNORLEN and software industry—ASSECO). WIG20 is an index of 20 most significant (liquid) companies traded on the

* Corresponding author. Tel.: +48 607913042.
E-mail addresses: kdomino@polsl.pl (K. Domino), tomasz.blachowicz@polsl.pl (T. Błachowicz).

http://dx.doi.org/10.1016/j.physa.2014.06.083
0378-4371/

Warsaw Stock Exchange [16]. The shares prices of chosen banks were used because of their similarities which are expected to reflect in shares prices interactions (the PEKAO bank was concerned here as a dominant company in the banking system). Additionally companies from other than banking branches were used since investors often hold shares of companies from different industries to decrease their correlation and risk of simultaneous high drops in values.

Let us look at the investigation methodology in a more precise way. Fractal dimension and the anomalous diffusion concept were investigated to reveal auto-correlation signatures of financial data [4,7–11]. Those auto-correlation signatures affect the Hurst exponent value calculated using the local DFA and cause it to drop below the 0.5 value. This is an anti-correlation signal and can be used to predict possible high drops of shares prices and rates of return. Copula functions were used to construct the multidimensional frequency distribution of data [3]. Furthermore, they were used to calculate the probability of extreme events e.g. high drops of the bivariate variable composed of shares prices of two banks. Shares prices were investigated in the period from 10.11.2004 (when the trading of PKOBP begun) to 02.01.2014. Their values are presented in Fig. 1.

2. The measurement of auto-correlations of shares prices using detrended fluctuation analysis

In this paragraph the method based on the analogy between a complex dynamical system and financial data is discussed [4–6]. The current method refers to the local property of the financial series and the local Hurst exponent which is sensitive to the anti-correlation signature and hence predicts possible high drops in shares prices and rates of return [7–14]. The local Hurst exponent is calculated for each share's price using the local Detrended Fluctuation Analysis (DFA) [7–12].

To perform DFA the observation window $N = 500$ corresponding to series of N sequent shares prices, is chosen. Let us define a share's price at the time t as $P(t)$. Here the observation window starts at $P(t)$ and ends at $P(t + N - 1)$. Inside the observation window the series is renumbered: $P(t) = P(m = 1), \ldots P(t + N - 1) = P(m = N)$ and divided into n non-overlapping sub-series of length $t_n = \lceil N/n \rceil$, with $\lceil\rceil$ being the ceiling function, returning the smallest integer no smaller than an argument [17]. For each sub-series the linear regression is performed and values of shares prices predicted by the regression (the vertical coordinate obtained from a linear regression) are denoted by $r^{(n)}(m)$. The detrended variance $F^2(n)$ is calculated for every sub-series—$P^{(n)}(m)$ as follows:

$$F^2(n) = \frac{\sum_{m=i}^{m=i+t_n} \left(r^{(n)}(m) - P^{(n)}(m)\right)^2}{t_n}. \tag{1}$$

The summation index m goes from i, at the beginning of the nth sub-series, to $i + t_n$ at the end of the nth sub-series. Next values of the detrended variance are averaged over n sub-series in order to obtain $\langle F^2(n)\rangle$, the averaged detrended variance, and the basic DFA relation $\langle F^2(n)\rangle \propto t_n^{2H}$ being a dependence between $\langle F^2(n)\rangle$ and time is obtained [18]. The linear regression of the logarithm is performed to determine a Hurst exponent as a slope. During calculations the accuracy of Hurst exponent fitting does not exceed 5.6% or 0.022.

The local Hurst exponent exhibiting values $H < 0.5$ appeared to be a significant indicator of the incoming crash or change in the trend of financial data [19,20]. To explain this behavior let us recall [21,22] that for $H = 0.5$ the system displays no long-term auto-correlations. For $H < 0.5$ it displays negative auto-correlations, and is more predictable than for the $H < 0.5$ case.

One can see that for PEKAO data the Hurst exponent values are variable and lower than 0.5 for all data, indicating negative auto-correlations (Fig. 2). For other banks the Hurst exponent values are higher (they increase as the capitalization and liquidity of the company decrease). For smaller banks they are sometimes exceeding the value of 0.5, evolving toward super diffusive, deterministic behavior. Low liquidity of some shares may disturb the arbitrage phenomenon which removes auto-correlations [4]. For nonbank companies ASSECO, PKNORLEN as well as KGHM display a similar variation of Hurst exponent values, despite the difference of industries under investigation.

3. The rates of returns analysis using probability frequency distributions

To compare the shares prices of different companies the percentage of return rates R_i was introduced in the following way:

$$R_i = \frac{P_{i+1} - P_i}{P_i}, \tag{2}$$

where P_i and P_{i+1} are the subsequent shares prices. Our intention was to analyze the negative threshold value $\alpha = -1\%$ defined as a low event and to pick return rates such as $R_i < \alpha$. Such a value of α was chosen, because frequently occurring respectively small drops of return rates may be a signal of price consolidation, log-periodic oscillations, anti-correlation signal and incoming long-term decreasing trend [4,7]. Such signals were detected before the maximum of the WIG20 index, recorded at the Warsaw Stock Exchange on 29th October 2007 [20]. Begun to decrease, the index lost 66% of its value during 328 trading days after the maximum.

Such events can be clearly determined with the use of the bivariate return-rate-variable $(R_i^{(1)}, R_i^{(2)})$, assuming that both $R_i^{(1)} < \alpha$ and $R_i^{(2)} < \alpha$ inequalities hold, for a given trading day (marked as i). We read out 403 such events for the first