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# Maximum entropy model for business cycle synchronization

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## HIGHLIGHTS

- We use the maximum entropy model to study business cycle synchronization of G7 system.
- We obtain the pairwise-interaction network of G7 system.
- The network shows the clustering structure which is associated with the region.
- The pairwise interactions account for 45% of the entire structure of the G7 system.
- The larger the system size is,the more important higher-order interactions become.

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## ABSTRACT

The global economy is a complex dynamical system, whose cyclical fluctuations can mainly be characterized by simultaneous recessions or expansions of major economies. Thus, the researches on the synchronization phenomenon are key to understanding and controlling the dynamics of the global economy. Based on a pairwise maximum entropy model, we analyze the business cycle synchronization of the G7 economic system. We obtain a pairwise-interaction network, which exhibits certain clustering structure and accounts for 45% of the entire structure of the interactions within the G7 system. We also find that the pairwise interactions become increasingly inadequate in capturing the synchronization as the size of economic system grows. Thus, higher-order interactions must be taken into account when investigating behaviors of large economic systems.



## 1. Introduction

Since the sub-prime mortgage crisis of the United States erupted, all major economies in the world have been inflicted with a severe financial crisis. Indeed, the global economy has experienced the worst recession since the Great Depression of the 1930s. This has in turn prompted an increase of academic interest in global business cycle [1,2].

Global business cycle can be characterized by simultaneous recessions or expansions of major economies; such dynamical similarity along business cycles is also called business cycle synchronization in the economics literature [3]. And there is quite an extensive literature in this research area. Frankel and Rose presented empirical evidence that higher bilateral

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trade between two economies is associated with more correlated business cycles [4]. Imbs stressed the linkage between similarity in industrial structure and business cycle synchronization in her paper [5]. Rose and Engel discussed the role of currency unions in business cycle synchronization by empirical analysis [6]. While these researches identified the factors that affect the degree of synchronization between economies, they did not, however, address the synchronization of the overall economic system.

The key to understanding the mechanism of synchronization is to uncover the interaction structure among economies [3]. The most common way of estimating the network structure of a complex system is to characterize the connection between elements by means of correlation coefficients. However, recent researches have shown that such characterization does not accurately estimate the network structure due to significant indirect correlations [7,8]. We argue that a more effective and informative approach is to derive the network of interaction based on the principle of maximum entropy.

The principle of maximum entropy as an inferential tool was originally introduced in statistical physics by Jaynes [9–11] and was further developed by other physicists afterwards [12–15]. Generally, observed signals of any given system are governed by, and therefore are manifestation of, the underlying structure of the system. The principle of maximum entropy provides a simple way by which we can infer the system's least-biased structure capable of generating these signals. Compared with the correlation coefficient, the approach succeeds in inferring interactions, from which it reconstructs correlations at all orders, and thus can estimate the network structure more accurately [7,8]. Due to its universality, the approach has been successfully applied to researches in ecology [16–19], life sciences [20–23], and neuroscience [7,24,25], among other disciplines. In particular, it has been shown that only pairwise interactions are sufficient to describe such complex systems as tropical forests [17], proteins [23], and retinas [7]. In this paper, we apply the principle of maximum entropy, built on pairwise interactions, to the business cycle synchronization of the seven most-developed economies in the world, known as G7.

## 2. Data

The data in this study are taken from the database OECD.Stat, where quarterly real GDPs of every member of OECD are available. The GDPs are calculated in terms of US dollars, adjusted by fixed PPPs (Purchasing Power Parity). The time period with available date for most countries is from 1960s first quarter to 2009s first quarter (amounting to 197 quarters). The total number of data points of all members is 5190 observations.

In order to apply a pairwise maximum entropy model, the data need to be converted into a binary representation—recession or expansion, in this case. To this end, we first calculate the average growth rate for each economy. Suppose the available data of GDP for an economy last over $N$ quarters, and the growth rate in the $i$th quarter is $r_i$, the average growth rate $\bar{r}$ can be obtained from the following relation:

$$\prod_{i=1}^{N-1}(1+r_i) = (1+\bar{r})^{N-1}. \tag{1}$$

We then define recession and expansion: if growth rate is less than the average growth rate, we define the state as recession and set the value of state variable to 1; otherwise, we define the state as expansion and set the value of state variable to 0.

The size of the system under consideration is limited by the data availability: in order to obtain reliable estimates of the parameters, the number of all possible states of the system should be well below the number of observations, i.e., $2^N < 197$. The G7 economic system is a small, yet meaningful, sub-system of the global economy. Its synchronous behavior can influence the business cycle of the global economy. As such, it is an excellent case study for our approach.

## 3. Principle of maximum entropy

The first step in the analysis with the principle of maximum entropy is to determine some meaningful constraints that describe the observed signals generated by the system. We then determine the least-structured distribution subject to those constraints. It is possible to prove that the Shannon entropy is the correct measure of the structure whose maximization, under a given set of constraints, would lead to the least-structured distribution [11].

Consider an economic system consisting of $N$ economies. We build a binary representation of the economic state by assigning a binary variable $\sigma_i$ to economy $i$: $\sigma_i = 1$ if economy $i$ is in a recession, and $\sigma_i = 0$ if the economy is in an expansion. Then a state for the whole economic system can be denoted by a vector $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_N)$. Our goal is to calculate the probability distribution $p(\boldsymbol{\sigma})$ that maximizes Shannon entropy

$$H = -\sum_{\boldsymbol{\sigma}} p(\boldsymbol{\sigma}) \ln p(\boldsymbol{\sigma}) \tag{2}$$

with the following constraints:

$$\sum_{\boldsymbol{\sigma}} p(\boldsymbol{\sigma}) = 1, \tag{3a}$$

$$\langle \sigma_i \rangle = \sum_{\boldsymbol{\sigma}} p(\boldsymbol{\sigma})\sigma_i = \frac{1}{T}\sum_{t=1}^{T} \sigma_i^t, \tag{3b}$$