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Stochastic resonance in a linear system with random damping parameter driven by trichotomous noise

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HIGHLIGHTS

- We investigate the stochastic resonance in a second-order linear system subject to trichotomous noise.
- The output spectral amplification (SPA) is a non-monotonic function of the coefficient of the trichotomous noise.
- The SPA is a non-monotonic function of the frequency of the driving signal.
- The SPA varies non-monotonously with the damping coefficient and the oscillator frequency.

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ABSTRACT

The stochastic resonance (SR) in a second-order linear system driven by a trichotomous noise and an external periodic signal is investigated. By the use of the properties of the trichotomous noise and the Shapiro–Loginov formula, the exact expression for the output spectral amplification (SPA) of the system is obtained. The non-monotonic influence of the coefficient of the trichotomous noise on the SPA is found. It is shown that the SPA is a non-monotonic function of the amplitude, the correlation rate and the probability of the trichotomous noise. The SPA varies non-monotonically with the frequency of the driving signal, the damping coefficient and the frequency of the linear system.

1. Introduction

The phenomenon of stochastic resonance (SR) characterizes the cooperative effect between a weak signal and noise in nonlinear systems, which was proposed as a plausible mechanism for the switch of the Earth's climate between ice ages and periods of relative warmth [1,2]. The SR phenomenon has been investigated in a variety of nonlinear systems with additive and multiplicative noise [3–12]. It is concluded that nonlinearity, periodic and random force are the three essential ingredients for the onset of SR, while for linear systems, it was suggested that noise multiplicativity and time correlation are the necessary conditions for the SR to occur.

The effects of dichotomous noise on linear systems have been studied by many researchers [13–18]. Duo to its including all cases of dichotomous noise, and more flexibility to model natural colored fluctuation, trichotomous noise is more useful than dichotomous in actual application. The effects of trichotomous noise on lots of dynamic system were considered. For example, Brownian particles in a spatially periodic asymmetric potential (ratchet) [19], and in a piecewise linear spatially periodic potential [20] have been investigated. The first two moments and the correlation function [21], the influences

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of noise flatness and friction coefficient of a harmonic oscillator [22], as well as the first moment for the output signal of a fractional oscillator [23] with trichotomous—noise-fluctuated frequency were also considered. On the other hand, quadratic noise exists actually in physical systems [24–29]. Many stochastic systems with quadratic noise have been investigated, including a stochastic differential equation with a quadratic nonlinearity in the noise [24], a one-dimensional evolution equation with a quadratic Smoluchowski noise [25], a linear first-order equation with a quadratic colored noise [26], a linear differential equation with an additive quadratic noise [27], a multidimensional system driven by a no-Markovian quadratic noise [28], and SR in a mono-stable system subject to quadratic colored noise [29].

A second-order linear system, i.e., a harmonic oscillator, is a simple model for different phenomena in Nature, whose dynamic property is a long-explored area going back to Galileo [30,31]. A noisy oscillator was introduced to science by Einstein 100 years ago in studies of Brownian motion. The first two moments and the correlation function [21,23], the influences of noise flatness and friction coefficient of a harmonic oscillator [22], with trichotomous noise fluctuating frequency was considered. On the other hand, there are an increasing number of problems where the damping parameter of a linear system is fluctuated. For example, particles advected by the mean flow passes through the region under study, including problems of phase transition under shear [32], open flows of liquids [33], Rayleigh–Benard and Taylor–Couette problems in fluid dynamics [34], dendritic growths [35], chemical waves [36], and motion of vortices [37]. In these cases, the velocity, i.e., the damping parameter entering the convective term is subject to fluctuations. However, to the best of our knowledge, little attention has been focused on the stochastic resonance in a linear system with random damping parameter driven by quadratic trichotomous noise. Thus, in this work, we aim to study the SR in a linear system with trichotomous-fluctuated damping parameter. Furthermore, we will show that the spectral amplification exhibits a SR behavior versus the damping coefficient, the driving frequency and the oscillator frequency. Thus the values of SPA can be controlled, i.e., either enhanced or suppressed, by changing the coefficient of the quadratic trichotomous noise, the damping coefficient, the driving frequency and the oscillator frequency.

The structure of the paper is as follows. In Section 2 the model investigated is presented. The exact formula for the SPA is found. In Section 3 the nonlinear dependence of the SPA on the trichotomous noise and the system parameters is analyzed and discussed, and finally some brief conclusion remarks are given.

2. The linear system and its spectral amplification

Consider a second-order linear system (i.e., a linear oscillator) with random damping parameter described by the following stochastic differential equation:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+[2r+\xi(t)+b\xi^2(t)]\frac{\mathrm{d}x}{\mathrm{d}t}+\omega^2x=A\cos(\Omega t), \tag{1}$$

where $x(t)$ is the oscillator displacement, r and ω denote the damping parameter and frequency of the linear oscillator (1), respectively. b denotes the coefficient of the quadratic noise, $\xi^2(t)$. The noise $\xi(t)$ is a trichotomous one, consisting of jumps between three values, i.e., $\xi(t)\in\{-a,0,a\}$, $a>0$. The jumps follow, in time, the pattern of a Poisson process, the values occurring with the stationary probabilities $p_s(a)=p_s(-a)=q$ and $p_s(0)=1-2q$, where $0<q<1/2$. The mean and correlation functions of $\xi(t)$ are

$$\langle\xi(t)\rangle=0,\qquad\langle\xi(t+\tau)\xi(t)\rangle=2qa^2\mathrm{e}^{-\lambda\tau}, \tag{2}$$

where λ is the correlation rate. The flatness of the trichotomous noise is

$$\kappa=\frac{\left\langle\xi^4(t)\right\rangle}{\left\langle\xi^2(t)\right\rangle}=\frac{1}{2q}. \tag{3}$$

In this work, we restrict ourselves to the case where for all states of the trichotomous noise, the damping coefficient is positive. We point out here that for $b=0$ and $\xi(t)$ being a dichotomous noise, our model is the same as those studied in Refs. [15,31].

To obtain the first moment of x, the well-known Shapiro–Loginov procedure [38] is applied, i.e., for an exponentially correlated noise $\xi(t)$, one has

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\xi m\rangle=\left\langle\xi\frac{\mathrm{d}m}{\mathrm{d}t}\right\rangle-\lambda\langle\xi m\rangle, \tag{4}$$

where m is an arbitrary function of the noise, $m=m(\xi)$. Eq. (1) can be corrupted as two first-order equations

$$\frac{\mathrm{d}x}{\mathrm{d}t}=y, \tag{5}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=-2ry-\xi y-b\xi^2y-\omega^2x+A\cos(\Omega t). \tag{6}$$