Physica A xx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Multistability and transition to chaos in the degenerate Hamiltonian system with weak nonlinear dissipative perturbation

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HIGHLIGHTS

- We study the effect of nonlinear dissipation on a system with the stochastic web.
- The scenario of attractors evolution as nonlinear dissipation increases is revealed.
- We observe a period-doubling cascade for the 1:3 resonance of natural frequency and pulse.
- Only hard transitions to chaos exist for all other resonances.

ARTICLE INFO

Article history:
Received 25 November 2013
Received in revised form 31 March 2014
Available online xxxx

ABSTRACT

The effect of small nonlinear dissipation on the dynamics of a system with the stochastic web which is linear oscillator driven by pulses is studied. The scenario of coexisting attractors evolution with the increase of nonlinear dissipation is revealed. It is shown that the period-doubling transition to chaos is possible only for the third-order resonance and only hard transitions can be seen for all other resonances.

0. Introduction

It is well known that dissipative and conservative systems demonstrate dramatically different types of dynamics. In particular, chaos in the non-integrable conservative systems can be observed for almost any parameters, but typically in a very narrow region of the phase space. Otherwise, in dissipative systems chaos appears only in the specific range of parameters but the basin of the chaotic regime usually is rather large [1–4]. If one introduces a small dissipative perturbation into a conservative system it goes into a specific "borderline" state, where both features of conservative and dissipative dynamics should be observed in some way. The peculiarities of systems with weak dissipation including the coexistence of large number of regular attractors and the scenarios of transition to chaos were studied in several recent works [5–16].

But most of these works consider the systems which are non-degenerate in the sense of KAM theorem [4]. However, it is well known that the structure of degenerate system's phase space differs significantly [1,2]. Zaslavsky [1,2] shows that in driven degenerate systems the resonance region covers the whole phase space for arbitrary small perturbation due to the independence of natural frequencies on the action variables. So the "web" of destructed separatrixes covers the phase space

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http://dx.doi.org/10.1016/j.physa.2014.05.066
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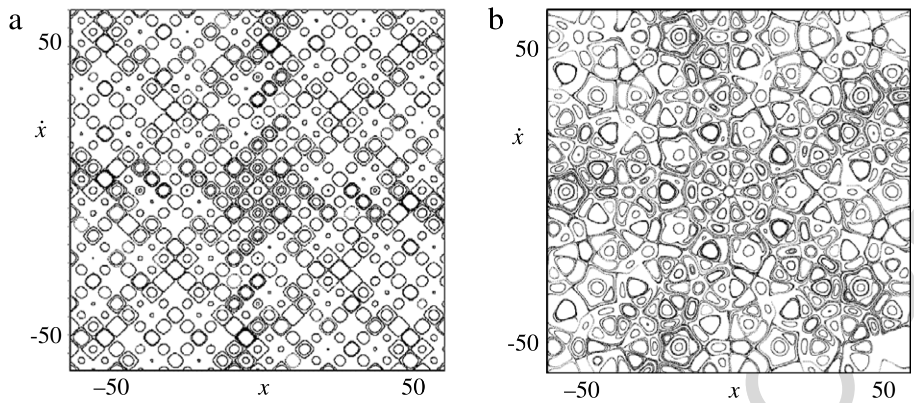

Fig. 1. Phase portraits of the system (1) with different parameter values q: $q = 4$ (a); $q = 5$ (b). The amplitude perturbation is $K = 0.1$.

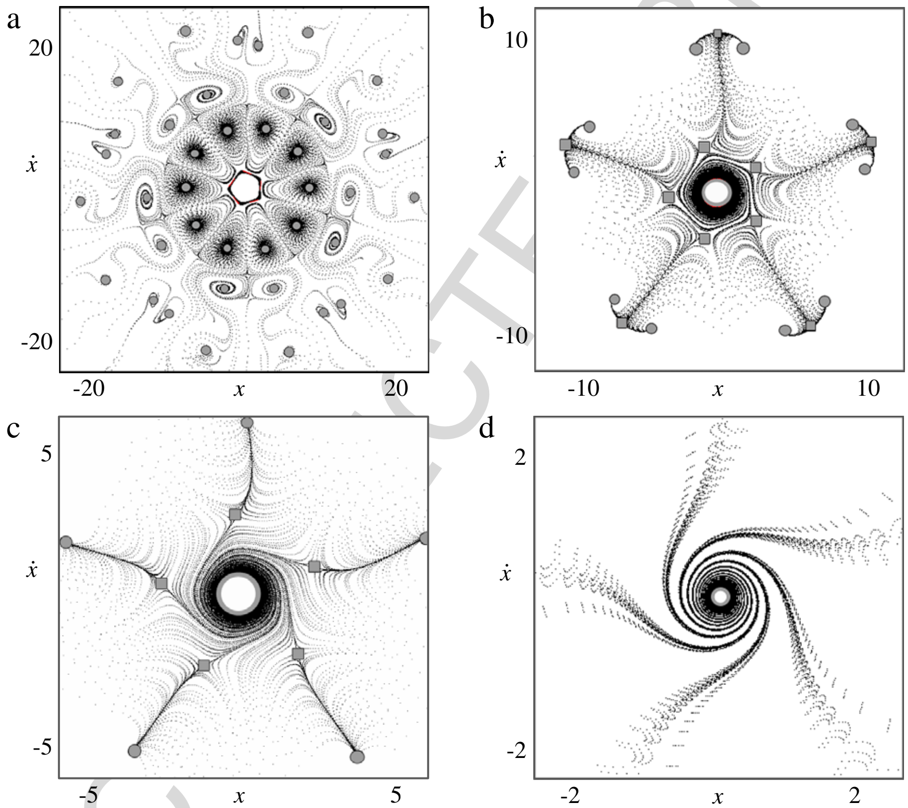

Fig. 2. Phase portraits of the system (2) for $q = 5, K = 0.1, \gamma = 1 \cdot 10^{-4}$ and different values of the parameter μ: $\mu = 5 \cdot 10^{-5}$ (a); $\mu = 5 \cdot 10^{-4}$ (b); $\mu = 2 \cdot 10^{-3}$ (c); $\mu = 5 \cdot 10^{-2}$ (d). The circles denote stable fixed points, the squares — unstable.

and forms the structures with rotational and (for some orders of resonance) translation symmetries which results in the unlimited diffusion in the radial direction (i.e., the unlimited growth of action) at any small amplitude perturbation. More detailed discussion and some examples of such structures can be found in Ref. [2]. In spite of rather high degeneracy such systems occur in several physical problems mainly concerning the motion of charged particles in the electromagnetic fields (see Ref. [17] for example). So the effect of small dissipative perturbation on the structure of the phase space seems to be interesting both from theoretical and physical points of view.

Previously there have been some studies on the effect of the fixed (linear) dissipation [18,19]. In the present work we study the effect of nonlinear (Van der Pole-like) dissipation on the structure of the stochastic web. One aim of the work is to reveal the scenarios of attractor's evolution when changing the nonlinear dissipation. Other is to reveal the scenario of