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A quantitative criterion to describe the deformation process of rock sample subjected to uniaxial compression: From criticality to final failure

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HIGHLIGHTS

- A renormalization group model is developed with a new stress transfer mechanism.
- The crack damage stress threshold, σ_{cd}, is equivalent to a phase transition point.
- The σ_{cd}/σ_{ucs} ratio may be an intrinsic property of low-porosity rocks.
- A quantitative criterion for the failure prediction of rock samples is established.

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ABSTRACT

It was found that the crack damage stress threshold, σ_{cd}, is a phase transition point during the deformation process of rock and is equivalent to the unstable fixed point of renormalization group theory. Thus, a relationship exists between the renormalization group theory and the rock deformation process, through which we can investigate the critical behavior of rock deformation. Therefore, an improved renormalization group model, which takes into account the different stress transfer mechanisms that are closer to the actual mechanical processes, is introduced to reveal the critical behavior of the rock deformation process. Finally, a quantitative relationship between the crack damage stress threshold and peak strength, $\sigma_{critical}/\sigma_{peak}$, is theoretically established. To test the theoretical relationship we additionally present experimental results of an investigation of the ratio of the crack damage stress threshold to uniaxial compressive strength, σ_{cd}/σ_{ucs}, based on different rock types. The results show that the overall average and standard deviation of σ_{cd}/σ_{ucs} is $0.80(\pm 0.10)$ for low-porosity igneous, metamorphic and sedimentary rocks, a figure that is closer to the theoretical solution of $\sigma_{critical}/\sigma_{peak}$ from the improved renormalization group model with a stress transfer mechanism of $STM - 1/r^3$. Our study implies that the $\sigma_{critical}/\sigma_{peak}$ ratio may be intrinsic to low-porosity rocks, and therefore can be considered as a reliable predictor of the peak strength of rock samples in the laboratory.

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1. Introduction

A quantitative evaluation of damage to rocks subjected to external loads has always attracted much interest predicting rock failure processes. Numerous studies have indicated that the deformation process of rock samples subjected to a uniaxial compression can be divided into five stages: (I) crack closure; (II) elastic deformation; (III) crack initiation and stable crack growth; (IV) crack damage and unstable crack growth; and finally (V) failure and post-peak behavior [1–4]. Moreover, three characteristic stress thresholds before the peak strength exist: crack closure stress, σ_{cc}, crack initiation stress, σ_{ci} and crack damage stress, σ_{cd}. The typical mechanical response described above has been plotted in Fig. 1 (the post-peak region is not shown for clarity). If a quantitative relationship between the peak strength and any one of the three characteristic stress thresholds can be found or constructed, the peak strength can be predicted in advance.

Numerous studies have indicated that the crack damage stress threshold, σ_{cd}, is critical in the deformation process [1–9]. When the external load is less than σ_{cd}, the crack growth is stable but once the load reaches or exceeds σ_{cd}, the crack propagation seems to be unstable and crack growth will not terminate even if the stress remains constant at σ_{cd}. Meanwhile, it has been found that macroscopic deformation of rock is often accompanied by changes in physical properties, such as permeability and resistivity [10–13]. Changes in these physical properties can generate abnormal responses around the crack damage stress threshold, σ_{cd}, where a significant increase in permeability and a clear decrease in resistivity can be relatively easily observed (Fig. 2), just as the percolation model has shown [14]. Essentially, the responses of permeability and resistivity can be attributed to these micro-structural changes. These abnormal features have been regarded as precursory signals of strong earthquakes [15].

As mentioned above, a rock sample essentially undergoes a phase transition at the crack damage stress threshold, as shown in Fig. 3. It is well known that the renormalization group (RG) theory is a powerful tool to resolve self-organized processes and the critical phenomena and to reveal the universality of the disorder system at the phase transition point with neglecting some trivial features [16,17]. Therefore, in the present paper the RG theory is adopted and improved to study the behavior at the crack damage stress threshold, σ_{cd}, and then establish a quantitative correlation between σ_{cd} and peak strength.

In Section 2 of this paper, we present a detailed description of a new RG theory model with a different stress transfer mechanism and establish a quantitative relationship between the crack damage stress threshold and the peak strength. Experimental verification is presented in Section 3, with conclusions drawn from our study presented in Section 4.

2. Theoretical study of correlation between the crack damage stress threshold and peak strength

In the field of geology and geophysics, some critical phenomenon have been reported and studied based on renormalization group theory, such as seismology [18–24], hydromechanics [25,26], rock mechanics [18,27–29] and the study of landslides [30], and a comprehensive introduction was conducted by Turcotte [31]. It is worth mentioning that a different RG model with consideration of a stress transfer mechanism between broken and unbroken rock elements was proposed by Smalley et al. [22], by which a number of interesting features of the stick–slip behavior of faults were revealed. In our paper, the RG model outlined by Smalley et al. [22] is adopted, but we introduce a new and different stress transfer mechanism.

2.1. Two-dimensional renormalization group model of rock sample

It is well known that as an applied load to a rock specimen gradually increases, an increasing number of microscopic cracks will coalesce around the potential macroscopic fracture plane until the macroscopic fracture occurs. Taking this simple premise, it is possible to consider the fracture process of rock samples as a two-dimensional RG model (Fig. 4) in which a macroscopic fracture plane is renormalized into many cells that form different order blocks, as shown in Fig. 4(c). The term "cell" in the first-order block denotes the smallest element that cannot be divided further.

The first-order block consists of four cells, while the second-order block consists of four first-order blocks. Similarly, the third-order block consists of four second-order blocks, and so on, essentially on an infinite scale.

There are five possible states for each block, namely B4U0, B3U1, B2U2, B1U3, and B0U4 (Fig. 4(d)). The letter "B" denotes broken cells or blocks (colored box), followed by the corresponding number of broken cells or blocks, while the letter "U" denotes unbroken cells or blocks (white box), also followed by the corresponding number of unbroken cells or blocks. Assuming that each cell may be either broken, with probability p_1, or unbroken, with probability $1 - p_1$, it is possible to compute the probability of the state of the first-order block. Based on the concept of permutation and combination, the probability of all four cells within the first-order block B4U0 being broken is p_1^4 and the probability that in B0U4 all four cells are unbroken is $(1 - p_1)^4$. Similarly, the probabilities of B3U1, B2U2, and B1U3 being broken are $4p_1^3(1 - p_1)$, $6p_1^2(1 - p_1)^2$, and $4p_1(1 - p_1)^3$, respectively.

The RG model described above is the traditional RG model and does not consider the interaction among cells or blocks; in other words, cells or blocks are treated as independent of each other. From a rock mechanical point of view this is not ideal because a load supported by broken rock elements can be transferred to adjacent unbroken rock elements. As a result, a RG model that includes a stress transfer mechanism (STM) was developed by Smalley et al. [22], with the following assumptions:

(i) the strength of each individual cell obeys a Weibull distribution;