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Statistical complexity measures as telltale of relevant scales in emergent dynamics of spatial systems

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HIGHLIGHTS

- We provide a novel definition of complexity measure for spatial systems.
- The novel measure detects mesoscales.
- The novel measure detects regions transitioning from order to disorder.

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ABSTRACT

The definition of complexity through Statistical Complexity Measures (SCM) has recently seen major improvements. Mostly, the effort is concentrated in measures on time series. We propose a SCM definition for spatial dynamical systems. Our definition is in line with the trend to combine entropy with measures of structure (such as disequilibrium). We study the behaviour of our definition against the vectorial noise model of Collective Motion. From a global perspective, we show how our SCM is minimal at both the microscale and macroscale, while it reaches a maximum at the ranges that define the mesoscale in this model. From a local perspective, the SCM is minimum both in highly ordered and disordered areas, while it reaches a maximum at the edges between such areas. These characteristics suggest this is a good candidate for detecting the mesoscale of arbitrary dynamical systems as well as regions where the complexity is maximal in such systems.

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1. Introduction

The definition of complexity has recently seen major improvements. It has been acknowledged for a long time that simply accounting for information (Shannon or Fisher information, for instance) does not fully grasp the notion of complexity, since a perfect chaos maximizes information but it is actually not much more complex than perfect order. As Crutchfield noted in 1994, *Physics does have the tools for detecting and measuring complete order equilibria and fixed point or periodic behaviour and ideal randomness via temperature and thermodynamic entropy or, in dynamical contexts, via the Shannon entropy rate and Kolmogorov complexity. What is still needed, though, is a definition of structure and a way to detect and to measure it* [1].

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Seth Lloyd counted as many as 40 ways to define complexity, none of them being completely satisfactory. A major breakthrough came from the definition proposed by Lopez-Ruiz, Mancini and Calbet (LMC) [2]. Although not without problems [3,4], LMC's complexity clearly separated and quantified the contributions of entropy and structure. LMC measured structure through *disequilibrium*. Building on this proposal, Kowalski et al. [4], refined the definition of disequilibrium.

One should note that while entropy is a general concept that can be applied across a wide range of model families, this is not the case with measures of structure. With structure one needs to know what to seek. Most efforts to define disequilibrium focus on time series. The case of spatial 1D systems has been also considered [5].

We are interested, though, in a statistical complexity measure (SCM) for models with 2 or 3 spatial dimensions. This includes dynamical PDE-based models, such as the Navier–Stokes one, and Agent-Based Models (ABM), such as Collective Motion [6]. Here we use the adjective *dynamical* because the structures we are interested in are easily recognized (at least visually) by studying velocity fields. Other models of interest are static (i.e., not characterized by its velocity field, but rather from scalar quantities as density or spin). Examples of these models include PDE-based models such as reaction–diffusion, and Cellular Automata (e.g. Ising models).

Our hypothesis is that, for these systems, a good candidate for capturing the structural component in the definition of complexity is a correlation (of the velocity field in the dynamical cases, and of density or other scalar fields in the static ones). That is

$$C(s(\hat{x}, t)) = H(s(\hat{x}, t))D(s(\hat{x}, t)), \tag{1}$$

where H stands for entropy (Shannon's, Fisher's, or Kullback–Leibler's), and D is a correlation. $C(s(\hat{x}, t))$ represents the local statistical complexity measure, and $s(\hat{x}, t)$ is the state of the system at time t in position $\hat{x}$, characterized by some scalar or vector field.

A global SCM is recovered by integration over all the simulation domain Ω:

$$C(s(t)) = \int_{\Omega} \mathrm{d}\hat{x} C(s(\hat{x}, t)). \tag{2}$$

$C(s(t))$ is an extensive property. An intensive property is derived by defining

$$C_I(s(t)) = \frac{C(s(t))}{\int_{\Omega} \mathrm{d}\hat{x}}, \tag{3}$$

which is simply the average complexity field.

It is also important to acknowledge that the perception of complexity is deeply imbricated with the scale of measurement. Therefore, we should aim at measuring C at different scales. The idea of studying complexity as a function of scale is not new, as represented for instance in the concepts of *complexity profile* [7,8] and *d-diameter complexity* [9]. In comparison with these generic frameworks, we aim at exploring a definition that disentangles the contributions from entropy and structure and directly exploits the characteristics of the models under study (velocity fields and density fields in particular) as a proxy for structure.

Density fields (specifically chemical concentrations) as a proxy for *structural information* were studied in the case of reaction–diffusion models [10]. This work analysed the behaviour of structural information as both a spatial field and across scales. We expect this approach to be also fruitful when structure is characterized by velocity fields rather than density fields, and further enlightened by separating the contributions of noise and structure.

We aim to study the validity of this hypothesis by measuring $C(s(t))$ in numerical simulations of different models. In this context we define (i) the microscale, as the size of the simulation mesh (small enough to represent the microscopical regime characterized by chaotic dynamics), (ii) the mesoscale, as the scale at which a maximum of complexity is observed (typically characterized by the formation of turbulence, vortices, clusters, bands, flocks, etc.), and (iii) the macroscale, large enough to reach the hydrodynamical (or equivalent) limit, typically characterized by different phases and phase transitions (ordered or disordered, but not complex).

Representing these three scales in a single simulation is notoriously difficult in some cases (for instance, a Navier–Stokes scenario). The microscale and macroscale are separated by many orders of magnitude. In this case, the strategy should include different simulations for the distinct scales, adapting the pertinent equations to each scale and dynamics.

We find a simpler prospect in Collective Motion. Here the dynamics are characterized by three scales within a few orders of magnitude, and it is then amenable to a single simulation encompassing them all. In this preliminary study, we will focus our attention on Collective Motion.

2. Statistical complexity measures

Initial value problems with spatial dimensions belong fundamentally to two big families: mesh-based and meshless. The former includes lattice discretizations of continuous PDE-based problems, such as Finite Volume Methods or Finite Difference Methods. It includes, as well, problems directly defined on a lattice, such as Cellular Automata (Ising models for instance). Meshless models include discretizations of PDE-based problems that use particle discretizations (such as Smoothed Particle Hydrodynamics), as well as problems directly defined on agents or particles, such as Collective Motion.

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