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Selective altruism in collective games

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HIGHLIGHTS

- We show the beneficial role of altruism in a modified version of Parrondo's collective games.
- We introduce selective altruism and study its diffusion in a community of agents.
- We show how the topology of the network influences the diffusion of altruist behaviour.

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ABSTRACT

We study the emergence of altruistic behaviour in collective games. In particular, we take into account Toral's version of collective Parrondo's paradoxical games, in which the redistribution of capital between agents, who can play different strategies, creates a positive trend of increasing capital. In this framework, we insert two categories of players, altruistic and selfish ones, and see how they interact and how their capital evolves. More in detail, we analyse the positive effects of altruistic behaviour, but we also point out how selfish players take advantage of that situation. The general result is that altruistic behaviour is discouraged, because selfish players get richer while altruistic ones get poorer. We also consider a smarter way of being altruistic, based on reputation, called "selective altruism", which prevents selfish players from taking advantage of altruistic ones. In this new situation it is altruism, and not selfishness, to be encouraged and stabilized. Finally, we introduce a mechanism of imitation between players and study how it influences the composition of the population of both altruistic and selfish players as a function of time for different initial conditions and network topologies adopted.

0. Introduction

Everyday we observe several examples of altruistic behaviour in the world around us, at different levels of complexity. Within a single organism, cells coordinate to keep control over their division and avoid the emergence of cancer. Structures that we observe in the organism (organs, systems) are the consequence of some kind of cooperative behaviour at the cellular level. Coordination and cooperation occur also within animal societies: for example, worker ants sacrifice their own fecundity and do not reproduce in order to serve their queen and colony, while, in a pride of lions, adult females nurse not only their own cubs but also those of other females. Humans help each other in many ways: we see small actions of cooperation in every day life, but also heroic acts like those of the workers of Fukushima nuclear power plant, who re-entered the contaminated building trying to bring things back under control.

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From the point of view of Darwinian evolution, it seems difficult to understand why such behaviour can exist and be so common. In fact, biological evolution is selection and struggle to survive and reproduce. Why should one help another individual, risking to lower his own reproductive success for the benefit of someone else? How cooperative and altruistic behaviour emerge and diffuse in nature?

Kin selection and inclusive fitness are a first possible explanation of these phenomena [1,2], even if restricted to forms of altruism towards close relatives, which are advantageous since they result in increasing the individual's genetic contribution to the next generation. Another possible mechanism able to answer the previous question lies in the key concept of repeated encounters [3]. If two individuals meet once with no chance of meeting again, the best choice for both is obviously to defect, since they have no reason to trust each other and face the risk to be betrayed. But, in a group where individuals often meet one another, the perspective drastically changes. If they help one another in the moments of need, all of them have an advantage. This is the basis of the so called "tit for tat" strategy, frequent in human and animal societies [4]. In this respect, there is also a strong evidence that natural selection operates among groups, as well as among individuals. It turns out that groups in which cooperating behaviour is present are favoured over groups of totally selfish individuals [5]. That is possibly why altruistic behaviour emerge and diffuse.

In the past years, altruism has also been analysed with the help of simple mathematical models and simulations, often in the context of game theory. Among the many studies going on in this direction, we may cite the work of Sigmund [6], Nowak [7], Gintis [8] and Helbing [9–11]. Along this line, in the present paper we focus on a collective version of Parrondo's games [12]. In particular, we consider here a variant of this model, developed by Toral [13], where we introduce altruistic and selfish players. We investigate how the action of altruistic players can create a positive condition for the whole community, but we also see how selfish players can take advantage of that situation, creating in the long run a negative condition for the community. In order to investigate how to prevent this bad outcome and explain the emergence of altruism in real situations, we introduce a new and more refined way of being altruistic, which we call *selective altruism*, inspired by the mechanism of indirect reciprocity. Then we explore through extensive numerical simulations the effects that it produces, also finding the conditions for its diffusion over the entire population.

The paper is organized as follows. In Section 1 we briefly recap Parrondo's paradox and some collective versions of Parrondo's games, focusing on Toral's one. In Section 2 we introduce the concept of altruism and elaborate a new specific model, called *Altruism–Selfishness (AS) model*, to take it into account. Then, in Section 3, in order to improve this naive altruistic strategy, we discuss a new model, the *Selective Altruism–Selfishness (SAS) model*, where we introduce a selective altruistic behaviour. In Section 4 we study the effects of imitation among players in a community of fully interacting individuals. Finally, in Section 5, we extend the previous analysis to several communities of players with different network topologies, comparing the simulation results. Conclusions and final considerations close the paper.

1. Parrondo's paradox in single and collective games

Parrondo's paradox [14,15,12] is a counterintuitive behaviour that takes place in the context of game theory and represents one of the numerous examples of systems where noise and randomness can play a beneficial role [16–23]. In particular, Parrondo showed that two losing games can result in a winning trend when played in an alternating or in a random sequence by a single player.

The games originally described by Parrondo are schematized in Fig. 1.

- *Game* A consists in a slightly biased coin, with a probability of winning that is less than one half (more precisely $1/2-\epsilon$).
- *Game* B, on the other hand, consists in two coins, a 'bad' one (with a winning probability of $1/10-\epsilon$) and a 'good' one (with a winning probability of $3/4-\epsilon$): the player tosses the bad coin if his capital is a multiple of three, otherwise he tosses the good coin.

As a consequence of game A or B, at each turn the player can win or lose a unit of capital competing against a casino. It can be proven that the two games, taken singularly, are fair if $\epsilon = 0$ and losing if $\epsilon > 0$. However, a player that alternates (periodically or randomly) the two games (starting from a zero capital) has, on average, a capital that increases with the number of turns, even for small positive values of ϵ. This apparent paradox has been explained by an analogy between the gambling game and a 1-dimensional Brownian motion under the action of a flashing potential [12]. Besides, the paradox can be explained from another point of view: the 'profitability' of game B depends on the probability π_0 that the capital is a multiple of three. It can be seen that game A lowers the value of π_0. In other words, game A reduces the number of times that the bad coin is used in game B, making it more profitable.

There are also collective versions of Parrondo's games that produce paradoxical results [24–29]. More specifically, in Refs. [24,25], the authors show that if every player chooses which game to play aiming only at his own profit, all the community ends up losing capital. On the other hand, in Ref. [26] it is shown that, if the players choose only for the immediate benefit of the community in the present turn, they still produce a losing trend. In those cases, it can be seen that choosing randomly between two games or two behavioural patterns avoids falling in the trap of an apparently optimal strategy and produces a winning result.

An interesting collective variant of this scheme, also producing paradoxical results, was introduced by Toral [13]. In the capital dependent version of this model (labelled as *version I'* in Toral's paper), depicted in Fig. 2, at each turn just one player P_i, in a community of N individuals $\{P_i\}_{i=1,2,\dots,N}$, is randomly selected for playing and he has to play one of two games. Game B

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