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On the control of opinion dynamics in social networks

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HIGHLIGHTS

- Opinion can be controlled by committed nodes which are immune to influence.
- We introduce conditions under which the opinion dynamics is controllable.
- Opinion fluctuation is determined by the smallest negative eigenvalue.
- Driver node with high K-shell can guide the network to the final position smoothly.

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ABSTRACT

This paper presents a framework to analyze the controllability of opinion dynamics in social networks using DeGroot model (DeGroot, 1974). We show how the opinion, or attitude about some common questions of interest in a population can be controlled by a committed node who consistently proselytizes the opposing opinion and is immune to influence. Some criteria are established to guarantee that opinion dynamics of networks can be perfectly or partially controlled. We also find that the opinion fluctuation is determined by the smallest negative eigenvalue of an influence matrix.

1. Introduction

Human behavior is profoundly affected by the influenceability of individuals and the social network that links them together. We base our study on an important model of influence network largely due to DeGroot [1]. In this model, the social structure of a society is described by a weighted and directed network. Each node in the network takes an initial *position* (between −1 and +1) about a common question of interest. At each date, nodes communicate with other nodes and update their positions because of influences from neighbors. The updating process is simple: a node's new position is the weighted average of his or her neighbors' positions from the previous period. Over time, positions may converge to a consensus, provided that some conditions are satisfied.

Intuitively, one can influence a set of nodes to guide the network's behavior towards a desired state. Let *an outside controller* be a node that can continuously influence its neighbors through the updating process, but never changes its own position. Suppose the initial position of the outside controller be +1 and we wish to bring the network to a final state, where all the other nodes' positions are +1 or positive. If all nodes in the network have the same final position with the outside controller, we say that the network is *perfectly controllable*. If all nodes' final positions are positive, then the network is

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partially controllable. We also call those nodes as *driver nodes* if they are directly influenced by the outside controller. We are particularly interested in identifying the minimum number of driver nodes, whose control is sufficient to control the network's opinion dynamics.

There is a large theoretical literature on social convergence and learning [2–6]. In contrast to the learning models, convergence and consensus in our work are analyzed to find ways that guide the network's behavior towards a desired state. The works of Refs. [7–9] are closer to the spirit of our work, but their models, questions, and basic structures are quite different from ours. Liu et al. [7] investigated the network controllability based on control and graph theories, whereas Wang et al. [8] adopted a pinning control strategy to drive a network from any initial state to a desired synchronous state. Kitsak et al. [9] addressed the issue of the identification of influential spreaders in networks. They found that the most efficient spreaders are those located within the core of the network. The spreading process models they apply are the susceptible–infectious–recovered (SIR) and susceptible–infectious–susceptible (SIS), which are always used to describe disease spreading as well as information and rumor spreading in social networks.

There is literature in physics and computer science on the DeGroot model and variations on it [10]. However, the focus has generally been on consensus [11,12] rather than on controllability. DeMarzo et al. examined a variation on the DeGroot model [13] where it allows updating to vary over time, so that an agent might place more or less weight on his or her own position over time. Another model [14] allows a node to only pay attention to other nodes whose positions are not far from his or her own. Thus, the model has a sort of distrust for information that is too different from his or her own. This is also closely related to the model [15] where at each time two agents are randomly matched and updating their positions only if the positions are close enough to each other. Yildiz et al. [16] investigated a model of discrete (binary) opinion dynamics based on a voter model. In this model, each individual holds one or two opinions. At each date, some randomly chosen individuals observe one of his or her neighbors and adopt the opinion. Authors characterized the effect of network structure and the opinion of stubborn agents on the long-run distribution of opinions. Another literature also discussed opinion dynamics that by applying linear exogenous control over DeGroot update cycles of beliefs, the agents can be persuaded to shift their beliefs in desired ways [17].

Indeed, although there is a large literature on social convergence and learning, there has been hardly any work on how to control the dynamics of continuous opinions under the DeGroot model. We address this by: (i) introducing the conditions under which the opinion dynamics of a social network is controllable; (ii) proposing general methods to control the opinion dynamics in social networks. We also examine what determines the fluctuation of opinions when the network reaches a consensus.

2. The DeGroot model

In the DeGroot model, a finite set $\mathbf{V} = \{1, 2, \ldots, n\}$ of nodes interact according to *an influence network*. The interaction patterns are represented by a n-by-n nonegative *influence matrix* $\mathbf{T} = \{T_{ij}\}_{i,j=1}^{n}$, where $T_{ij} > 0$ defines the degree of influence that node j has on the position of node i. Nodes update positions by repeatedly taking weighted averages of their neighbors' positions with T_{ij} being the weight that node i places on the current position of agent j in forming his or her position for the next period. Each node, say i, has a position $p_i^{(t)} \in [-1, +1]$ at time t. The updating rule is

$$\mathbf{p}^{(t)} = \mathbf{T}\mathbf{p}^{(t-1)},$$

and so

$$\mathbf{p}^{(t)} = \mathbf{T}^t\mathbf{p}^{(0)}.$$

The matrix $\mathbf{T}$ represents the social network of interactions, i.e., $T_{ij} = 0$ implies that node i does not get direct information from node j regarding his position, or equivalently, there is no directed link from node i to j in the underlying social network. The weight matrix $\mathbf{T}$ is a (row) *stochastic matrix*, i.e., the sum of entries across each row is equal to one. At each time instance nodes update their positions to a convex combination of their current positions and the positions of their neighbors. This process is reasonable and has many nice properties despite its simplicity [11,12].

For an influence network, its influence matrix $\mathbf{T}$ is *convergent* if $\lim_{t\to\infty} \mathbf{T}^t\mathbf{p}^{(0)}$ exists for all initial vectors $\mathbf{p}^{(0)} \in [-1, +1]^n$. Since the matrix $\mathbf{T}$ is an n-by-n row stochastic matrix, it can be regarded as the one-step transition probability matrix of a Markov chain with n states and stationary transition probabilities, and the standard limit theorem of Markov chain can be applied [1]. It can be shown that if the matrix $\mathbf{T}$ is such that the Markov chain with transition matrix $\mathbf{T}$ is *irreducible* and *aperiodic*, then nodes' positions reach a consensus in the limit. In particular, a stochastic matrix $\mathbf{T}$ is convergent if and only if every set of nodes that is strongly connected and closed is aperiodic [10]. If there is more than one strongly connected closed set, the influence network will not always reach a consensus.

3. Influence controllability

We consider a society envisaged as an influence network $\mathbf{G}(\mathbf{V}, \mathbf{E})$ of n interacting nodes, communicating and exchanging information. At time $t = 0$, each node $v \in \mathbf{V}$ starts with an initial position $p_v^{(0)} \in [-1, +1]$. *An outside controller* never changes his position: he might correspond to an opinion leader or a political party wishing to influence the rest of the

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