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Detection of anyon's braiding and identification of anyon entangled states in optical microcavities

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HIGHLIGHTS

- Simulate anyon in a composite system of microcavities and quantum dots.
- Dynamically detect braiding operations of anyon for a minimum unit of four qubits.
- Discriminate anyon entangled states based on the above method.

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ABSTRACT

In this paper, a simulation of anyon states in a composite system of quantum dots and optical microcavities is introduced. We construct a novel quantum gate, the braiding detecting gate (BDG), by making use of interactions between the quantum-dot electron spins in optical microcavities and photons to detect the dynamic braiding operation of anyons. Additionally, by means of the BDGs, we also present a protocol to distinguish different entangled states of anyons.

1. Introduction

In the experimental investigations of the fractional quantum Hall effect [1,2], a kind of quasiparticle in two dimensions is discovered, namely anyon. This kind of quasiparticle is famous for its amazing properties [1,3–5]. Anyons obey fractional statistics, which is characterized by a fractional statistical parameter and belongs to intermediate statistics [6]. Therefore, studies on anyons have become an important branch of modern physics [7–27]. Generally, anyons of each type cannot be created singly but in pairs. When two different kinds of anyons braid, there will be an additional phase factor added to the wave function of the system. In recent years, based on these discoveries, several theoretical models of anyons were put forward. One of the exactly solvable models is the first Kitaev model [28,29], which has been experimentally realized in photonic [20] and NMR [27] systems. The original model is a honeycomb lattice model with spin-1/2 system. Through the perturbation theory, the model can be changed into a quadrangular lattice model with a spin on each edge of the quadrangle. The vacuum, e particle, m particle and $\varepsilon = e \times m$ are four superselection sectors of the first Kitaev model. When we use

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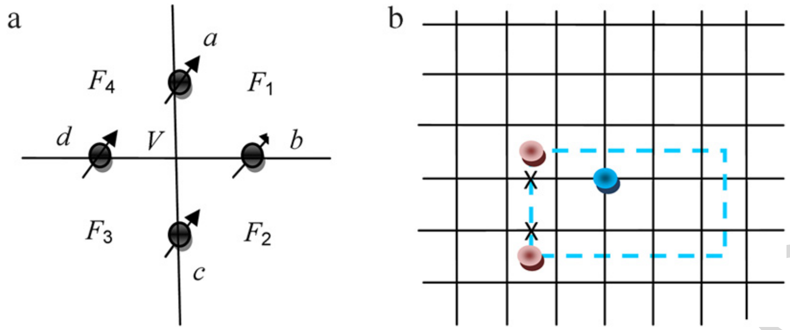

Fig. 1. (a) The minimum unit for anyon statistics. In the first Kitaev model, a, b, c, d are four spins. V is a vertex and F_1, F_2, F_3, F_4 are four faces. e and m particles are generated on the vertices and faces, respectively. (b) The braiding operation of anyons in the first Kitaev model is to move particles by means of applying Pauli operations on the spins in turn.

a Pauli Z operator on a spin laid on the edge, two e particles are created on the two vertices of that edge. When a Pauli X operation is applied on a spin, two m particles are generated on the two faces connected to the edge.

As a spin-1/2 system, electron spins are a natural candidate for realizing such anyon systems. A charged quantum dot in optical microcavities can create particles with negative charges. These particles are electrons restricted in one hole. When a photon interacts with these electrons, the whole system acts like a beam splitter in the limit of weak fields [30]. Since the photon–electron interaction will change the photon's state in a given way while unperturbing the spin's state, we can simulate the first Kitaev model and construct the braiding detecting gate (BDG) to detect the braiding operation in the composite system of four quantum dots and optical microcavities. Applying the BDG, we can dynamically monitor the process of braiding meanwhile undisturbing the system.

On the other hand, as known to all, quantum entanglement plays an important role in the quantum information processing. For example, it accelerates the computation of quantum computers [31–35], dense coding [36,37], quantum-state sharing [38–40] and quantum key distribution [41–45]. Entanglement purification [46–52] and concentration [53–58] are two kinds of ways to distill subsets of maximal entangled states. Because anyons are created in pairs, the quantum entanglement states of anyons are very common in reality. Therefore, it becomes urgent to distinguish different entanglement states of anyons in the quantum information processing of anyons. In this paper, we also put forward a method to distinguish different anyon entangled states utilizing the BDG realized in the above quantum-dot and optical-microcavity systems.

This paper is organized as follows. In Section 2, the anyon state is simulated in the composite system of quantum dots and optical microcavities and the BDG is constructed to observe the braiding operation. In Section 3, we propose a protocol to identify different types of anyons and their entangled states. Finally, the main points are summarized in the Conclusion part.

2. The braiding detecting gate

Recently, Xi and Hu proposed a new method to demonstrate anyon statistics [59], where only four qubits are required to create anyons and demonstrate the braiding operation (see Fig. 1(a)). The Hamiltonian of our system is

$$H = -A_v - B_{F_1} - B_{F_2} - B_{F_3} - B_{F_4}, \tag{1}$$

where $A_v = X_a X_b X_c X_d$, $B_{F_1} = Z_a Z_b$, $B_{F_2} = Z_b Z_c$, $B_{F_3} = Z_c Z_d$, and $B_{F_4} = Z_a Z_d$. The braiding operation where one anyon moves around another is one of the most distinct properties of anyons. When different kinds of anyons braid with each other, an additional phase factor appears in the wavefunction of the system. In the case of the first Kitaev model, the phase factor is -1. For example, when an m particle goes around an e particle for one circle, there will be a π phase in addition to the original wavefunction. On the other hand, when these two particles are of the same kind, e.g., two m particles or two e particles, the wavefunction remains itself after braiding. To realize the braiding operation in the first Kitaev model, Pauli X and Pauli Z operators are needed, cf. Fig. 1(b). e particles are created in pairs on the vertices of the quadrangle using the Pauli Z operator. Moving the e particle means applying Z operators on the spins sequentially. m particles are generated in pairs on the faces of the quadrangle through the Pauli X operator. Moving the m particle means applying X operators on the spins in turn.

In the first Kitaev model, by defining the single electron spin states $|\uparrow\rangle \equiv |0\rangle$ and $|\downarrow\rangle \equiv |1\rangle$, the braiding operation can be constructed as follows:

$$Z_a X_a X_d X_c X_b Z_a, \tag{2}$$

where $Z_a (X_a)$ is the Pauli $Z (X)$ gate on spin a (see Fig. 1). The BDG can show us dynamically the braiding operation between e particle and m particle.

We propose realizing the qubits by means of the quantum-dot electron spins in microcavities with $|\uparrow\rangle$ and $|\downarrow\rangle$ denoting the polarized spin states. When a photon passes through a microcavity, it interacts with the electrons. Inside the microcavity,

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