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Synchronizability in complex *ad hoc* dynamical networks with accelerated growth

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HIGHLIGHTS

- We investigate the synchronizability and topological periodicity of the dynamical networks with accelerated growth and ad-hoc property.
- The accelerated parameter is closely related to the synchronizability of the new model, as well as the non-periodicity of network topology.
- Preferential attachment plays a key role to enhance the synchronization of the new model and to easily change its non-periodical topology.
- Topological periodicity has robustness and fragility against random and specific removal of the nodes, respectively.

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ABSTRACT

Recent research works have been pursued in connection with network synchronizability under various constraints for different topological structures and evolving mechanisms. However, the fundamental question of how the synchronizability of the networks relates to accelerated growth and *ad hoc* property in the evolving processes remains underexplored. Here we study the *ad hoc* dynamical models with accelerated growth where the total number of edges increases faster than linearly with network size. By adopting three attachment mechanisms: random, rewired, and preferential attachment, we investigate the second-largest eigenvalues and the ratios of the extremal eigenvalues of coupled matrices in the accelerated models with different evolving parameters. For the new models, we demonstrate the robustness and fragility of synchronization against random and specific attacks by numerical simulations. We find that accelerated growth represents a convenient tool for improving the synchronizability of an evolving network. Furthermore, we show that not only network synchronization but also topological periodicity has robustness and fragility against random failures and specifical removal of the nodes, respectively. In particular, when *ad hoc* property is suggested in the evolving networks, we find that the deletion of nodes is easier to change network synchronizability and robustness compared to the addition of nodes.

1. Introduction

Complex networks widely abound in the real world. The internet, power grid, citation network, collaboration network, food web are all examples ranging from society, technological field to biological world, which consist of a lot of agents (or

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nodes) interacting with each other over those complex networked systems [1–3]. Since the discovery of two important characteristics: small-world [4] and scale-free properties [5], many researchers have paid strikingly increasing attention to the studies of complex networks and their extension, such as topological evolution, synchronization, information spreading, and percolation over some particular networks [2,3].

Among the main research issues on complex networks, the combined problems of network evolution and synchronization have received a great deal of attention from various fields of science and engineering in recent years [6–22]. In fact, an important purpose of studying complex networks is to understand the collective behavior of all nodes and edges. This particular behavior is difficult to explain in terms of the simple addition of the individual behavior of each node. Obviously, synchronization is possibly one of the simplest and most frequent behaviors. Salvo of flocks in summer evenings, simultaneous glow of fireflies, synchrony of myocardial cells are examples to show its ubiquitous characteristic [6]. Earlier works on synchronization of coupling networks focus on the completely regular or randomly coupling configuration [7,8]. However, when considering the complexity of network topological structure and evolving mechanism, one can find some significant differences of network synchronization compared with regular or random coupling networks. Ongoing research has recently focused on synchronization with small-world [9–12] and scale-free topological structures [13–15] in complex dynamical networks.

And more recently, the understanding of what mechanism controls the synchronization of dynamical networks has uncovered some particular evolving mechanisms ranging from rewired attachment in small-world networks to preferential attachment in scale-free networks. The changes in the network topology and evolution, such as small-world structure [16], degree correlation [17], and hub nodes [18] will distinctly enhance or weaken the network synchronization. In the evolving processes, some representative operations such as the deletion of some nodes [19] and discontinuous coupling strategy [20] have an obvious influence on coupling strength and synchronous time. Meanwhile, some research interests focus on the particular evolutionary mechanisms under the constraint of network synchronization. An amazing work in this aspect is to propose a new entangled network model with optimal topology [21]. And network growth under the constraint of synchronization stability has recently been discussed [22].

Since the topological characteristics and dynamical behavior of a complex network are manifested after its topological structure has evolved to a stabilized state, an interesting and significant question is how to synchronize in the whole evolutionary process. In this paper, we propose the *ad hoc* network models with accelerated growth for analyzing the functional relationships of network synchronization and topological periodicity in terms of acceleration parameter, addition probability of new nodes, and deletion probability of the existing nodes. Moreover, according to three attachment mechanisms of new adding edges: random, rewired, and preferential attachments, we explore the relevant factors of the attachment mechanism affecting network synchronizability. Finally, we discuss the robustness and fragility of network synchronization with the above-mentioned parameters. Those results allow us to draw novel relationships between network synchronization and the evolution mechanism including the adding and deleting rules of the nodes.

The organization of the remaining parts is as follows: in Section 2, the synchronizability of a complex dynamical network is defined and the ratio of the extremal eigenvalues of the coupled matrix for this model is suggested. The description of the *ad hoc* network model with accelerated growth and its corresponding evolving parameters are also given. In Section 3, numerical results about network synchronizability and topological periodicity for the new models with accelerated growth are derived. Additionally, the robustness and fragility of network synchronization with random and specific attacks of the nodes are investigated in Section 4. Finally, Section 5 concludes the whole paper.

2. *Ad hoc* dynamical network model with accelerated growth

2.1. Synchronizability of complex dynamical networks

Suppose that a dynamical network consists of N identical linearly and diffusively coupled nodes. The state equations of the coupled network whose each node has an n-dimensional dynamical system are

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j, \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \mathfrak{R}^n$ are the state variable of node i and the coupling strength is represented by the constant $c > 0$ [7,8,14]. We assume that $\Gamma = diag(r_1, r_2, \ldots, r_n) \in \mathfrak{R}^{n\times n}$ with $r_i = 1$ for a particular i and $r_j = 0$ for $j \neq i$. The coupling matrix $A = (a_{ij}) \in \mathfrak{R}^{N\times N}$ characterizes the coupling configuration of the network, and the elements of this matrix can be valued by $a_{ij} = a_{ji} = 1$ if there is an edge between node i and node j ($i \neq j$); otherwise, $a_{ij} = a_{ji} = 0$ ($i \neq j$). The diagonal elements of the coupling matrix are $a_{ii} = -k_i = -\sum_{j=1, j\neq i}^{N} a_{ij}$ where k_i is the degree of node i.

Dynamical network (1) is (asymptotically) synchronized if $x_1(t) = x_2(t) = \cdots = x_N(t) = s(t)$, as $t \to \infty$, where $s(t)$ is a solution of an isolated node [7]. Consider a connected network without isolated nodes or clusters; the coupling matrix of the network $A = (a_{ij})_{N\times N}$ is symmetric and irreducible. Let $0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N$ be the eigenvalues of A. As we know, given the dynamics of an isolated node, the coupling strength c characterizes the synchronizability of the network with respect to a specific coupling configuration A. And a small coupling strength implies a strong network synchronization