Sensors and Actuators A 168 (2011) 95–100

Contents lists available at ScienceDirect

Sensors and Actuators A: Physical

journal homepage: www.elsevier.com/locate/sna

Modeling and characterization of piezoelectric cantilever bending sensor for energy harvesting

R. Ly[a,b,*], M. Rguiti[a,b], S. D'Astorg[a,b], A. Hajjaji[c], C. Courtois[a,b], A. Leriche[a,b]

[a] Univ Lille Nord de France, F-59000 Lille, France
[b] UVHC, LMCPA, F-59600 Maubeuge, France
[c] ENSA, Univ Chouaib Doukkali, El-Jadida, Morocco

ARTICLE INFO

Article history:
Received 6 January 2011
Received in revised form 11 April 2011
Accepted 11 April 2011
Available online 15 April 2011

Keywords:
Energy harvesting
Piezoelectric cantilever
Bending sensor
Euler–Bernoulli Beam Theory
Analytical model

ABSTRACT

The main aim of this work is to enhance the conversion of mechanical energy into electrical energy by using direct piezoelectric effect. Under the assumption of the Euler–Bernoulli Beam Theory, a piezoelectric cantilever bending of 31-effect was developed. The equations of motion for the global system were established by using Hamilton's principle and solved by using the modal decomposition method. It provided the transfer functions model between the inputs (force) and the outputs (voltage) allowing the description of its dynamic behaviour for energy harvesting. The model was implemented by using Matlab software and will be able to integrate with the circuit model of energy storage. The results obtained show a good agreement with the experiments and other previous works. The model and the experiment indicate that the second mode of resonant frequency provides the voltage and the bandwidth much larger than the first mode. While the mass at the free end increases, the voltage obtained by the first mode increases. In contrast, the voltage obtained by the second mode decreases.

1. Introduction

The mechanical energy exists almost everywhere. The source can be a movement of human body, a vibrating structure surrounding a system or a wind and wave power. There are several techniques to convert them into electrical energy such as piezoelectric, electromagnetic and electrostatic transducer. The choice of the techniques depends on the source and its application. According to the literature, using direct piezoelectric effect is one of the favorite ways which well adapts to the applications of micro power suppliers. This energy conversion can replace the battery and be used as a power supply for autonomous wireless sensors and wearable electronics.

The aim of this paper is to convert efficiently the mechanical vibrations into electrical energy by using direct piezoelectric effect. Those mechanical vibrations of the environment usually occur with various frequencies. The piezoelectric converter provides the maximum energy conversion while its natural frequency is close to the frequency of mechanical source. For that reason, we proposed a mathematical model which can predict the natural frequency of the piezoelectric cantilever bending sensor and can describe its dynamic behaviour for a better energy conversion.

Many authors have investigated piezoelectric modeling with an analytical solution or a finite element approach, but most of them have intended to piezoelectric actuator/sensor modeling for vibration control of active structures [1,2]. Over the past few years, there has been an increasing attention on the use of autonomous wireless sensors and wearable electronics. For that reason, the authors have started to interest in experiments and modeling of a piezoelectric converter for energy harvesting which is used as autonomous microsystems suppliers. Poulin et al. [3] developed an analytical model of long piezoelectric bar 33-effect based on Newton's law. This model was presented under the form of the Mason's equivalent circuit and allowed to study the evolution of electrical power versus frequency. Ferrari et al. [4] proposed an experiment of a piezoelectric multifrequency energy converter made of three piezoelectric bimorph cantilevers with the same dimensions and different masses at the free end. As a consequence, each cantilever has different fundamental resonant frequencies which allow widening the overall equivalent bandwidth of the converter array. Under the assumption of the Kirchhoff plate theory, De Marqui et al. [5] presented an electromechanical finite element plate model which is based on the Hamilton's principle to establish the equation of motion. Many authors developed the model of piezoelectric converter, based on physical principles, in which the mechanical behaviour is described as a single mass spring-damper system

* Corresponding author at: Univ Lille Nord de France, F-59000 Lille, France. Tel.: +33 618910561.
E-mail addresses: lyrith@hotmail.com (R. Ly), mohamed.rguiti@univ-valenciennes.fr (M. Rguiti).

0924-4247/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.sna.2011.04.020

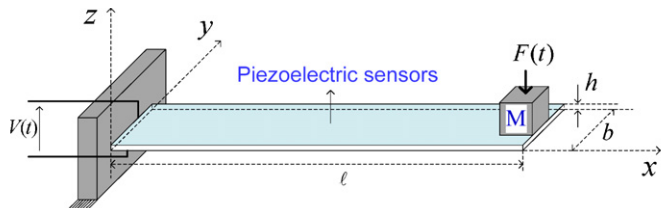

Fig. 1. Schematic view of piezoelectric cantilever.

[6–9]. However the single mass spring model allows describing the dynamic behaviour for only the first mode of resonant frequency. In case of the mechanical source happening at high frequency (vary from 0.5 to 2 kHz), the second and the following modes can be useful.

The main focus of this work is to model a piezoelectric cantilever bending of 31-effect for energy harvesting as shown in Fig. 1. Under the assumption of the Euler–Bernoulli Beam Theory (EBT), the equations of motion for the system were established basing on Hamilton's principle and solved by using the modal decomposition method. This method is extended from the previous work of longitudinal vibrations modeling [10,11]. This model allows providing the transfer functions between the inputs (for example, an external force $F(s)$) and the outputs (for example, a transverse displacement and a voltage $V(s)$) which can present the dynamic behaviour in difference kinds of the mode shapes by time and frequency response. The model was implemented using Matlab software and will be able to integrate with the circuit model of energy storage [12], for instance, developed by Shen et al. [7] and Ajitsaria et al. [13]. The results obtained by analytical solution are compared with the experimental ones.

2. Linear equations and variational formulation

The modeling and analysis of piezoelectric cantilever are made under the assumption of Euler–Bernoulli. The displacements u, v and w are denoted respectively along the coordinates x, y and z. Only the displacements in the plan Oxz are considered, the transverse displacements can be obtained:

$$v = 0, \tag{3-1}$$

$$w = w(x, t) \tag{3-2}$$

The axial displacement results from the rotation of the cross section:

$$u = -z\theta = -z\frac{\partial w(x, t)}{\partial x} = -zw_{,x}(x, t) \tag{3-3}$$

where θ is the slope. Throughout this paper, partial derivatives are denoted as:

$$f_{i,j} = \frac{\partial f_i}{\partial x_j},\ f_{i,jj} = \frac{\partial^2 f_i}{\partial x_j^2},\ f_{i,jjj} = \frac{\partial^3 f_i}{\partial x_j^3} \text{ and } f_{i,jjjj} = \frac{\partial^4 f_i}{\partial x_j^4}$$

Under the assumptions of the linear strain-displacement, the component axial S_1 along Ox of strain tensor can be simplified [14]:

$$S_1(x, z, t) = \frac{\partial u(x, z, t)}{\partial x} = -z\frac{\partial^2 w(x, t)}{\partial x^2} = -zw_{,xx}(x, t) \tag{3-4}$$

The width and thickness of piezoelectric cantilever are very small compared to its length. For that reason the components T_2 and T_3 of stress tensor are very small and they can be neglected. Under the assumptions of a piezoelectric cantilever 31-effect, the linear constitutive relations can be simplified as follows [15]:

$$T_1 = \underline{c}_{31}^E S_1 - \underline{e}_{31} E_3, \tag{3-5}$$

$$D_3 = \underline{e}_{31} S_1 + \underline{\varepsilon}_{31}^S E_3, \tag{3-6}$$

where $\underline{c}_{31}^E$, $\underline{e}_{31}$ and $\underline{\varepsilon}_{31}^S$ are respectively the components 31-effect of the elastic, the piezoelectric and dielectric constant, D_3 and $E_3 = -\phi_3(z, t)$ denote respectively the components of electric displacement and electric field and ϕ is the electric potential. The electric enthalpy density is obtained as:

$$H = \frac{1}{2}(\underline{c}_{31}^E S_1^2 - 2\underline{e}_{31} E_3 S_1 - \underline{\varepsilon}_{31}^S E_3^2) \tag{3-7}$$

According to the Hamilton's principle, the variation of the action between any two times t_1 and t_2 can be obtained as:

$$\delta\int_{t1}^{t2}(L + W)dt = 0 \tag{3-8}$$

where $L = \int_{\Omega_0}(T - H)d\Omega$ is the Lagrangian expression.

By definition, the kinetic energy is given by [11]:

$$\int_{\Omega_0} T d\Omega = \frac{1}{2}\int_0^\ell \rho_0 I\dot{w}_{,x}^2(x, t)dx + \frac{1}{2}\int_0^\ell \sigma\rho_0\dot{w}^2(x, t)dx \tag{3-9}$$

where $I = \int_\sigma z^2 d\sigma$ is the moment of inertia of the cross section, ℓ is the total length, σ is the cross-sectional area, Ω_0 is the volume of the piezoelectric cantilever at rest, ρ_0 is the mass density. The first term of Eq. (3-9) corresponds to the kinetic energy of the cross-sectional rotation and the second is the kinetic energy of the vertical translation. The virtual work W can be obtained by the sums of energies which are created by the electric surface charge $\bar{Q}(t)$, the mechanical point force $F(t)$ and the mass M at free end as:

$$W = -\int_{\Sigma_Q}\bar{Q}(t)V(t)d\Sigma + F(t)w(\ell, t) + \frac{1}{2}M\dot{w}^2(\ell, t) \tag{3-10}$$

where Σ_Q is the domain boundaries on which the surface charge are imposed. By replacing the various terms in Eq. (3-8), the expression of the Hamilton's principle is obtained as:

$$\begin{aligned}\delta\int_{t1}^{t2}\Bigg(&\frac{1}{2}\int_0^\ell \rho_0(I\dot{w}_{,x}^2(x, t) + \sigma\dot{w}^2(x, t))dx - \frac{1}{2}\int_0^\ell \underline{c}_{31}^E I w_{,xx}^2(x, t)dx \\ &+ \int_0^\ell\int_{-h/2}^{h/2} b\underline{e}_{31} z\phi_{,z}(z, t)w_{,xx}(x, t)dzdx + \frac{1}{2}\int_{-h/2}^{h/2} b\ell\underline{\varepsilon}_{31}^S\phi_{,z}^2(z, t)dz \\ &- \int_{\Sigma_Q}\bar{Q}(t)V(t)d\Sigma + F(t)w(\ell, t) + \frac{1}{2}M\dot{w}^2(\ell, t)\Bigg)dt = 0\end{aligned} \tag{3-11}$$

According to Fig. 1, one side of the piezoelectric cantilever located in the plan Oyz is fixed. This imposes the kinematic boundary conditions as follows:

$$w(0, t) = 0 \tag{3-12}$$

$$w_{,x}(0, t) = 0 \tag{3-13}$$

3. Analytical approach

Applying the variational principle to Eq. (3-11) with respect to the arbitrary variations of the vertical displacement δw and the electric potential $\delta\phi$, the dynamic equilibrium equations are obtained as:

$$-\rho_0 I\ddot{w}_{,xx}(x, t) + \sigma\rho_0\ddot{w}(x, t) + \underline{c}_{31}^E I w_{,xxxx}(x, t) = 0 \tag{4-1}$$

$$\phi_{,zz}(z, t) - \alpha w_{,xx}(x, t) = 0 \tag{4-2}$$

with $\alpha = -(\underline{e}_{31}/\underline{\varepsilon}_{31}^S)$. The equilibrium boundary conditions at $x = \ell$ and $z = \pm(h/2)$ are:

$$-\underline{c}_{31}^E I w_{,xxx}(\ell, t) + M\ddot{w}(\ell, t) + \rho_0 I\ddot{w}_{,x}(\ell, t) = F(t) \tag{4-3}$$