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Pinning synchronization of delayed complex dynamical networks with nonlinear coupling

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ABSTRACT

In this paper, we find that complex networks with the Watts–Strogatz or scale-free BA random topological architecture can be synchronized more easily by pin-controlling fewer nodes than regular systems. Theoretical analysis is included by means of Lyapunov functions and linear matrix inequalities (LMI) to make all nodes reach complete synchronization. Numerical examples are also provided to illustrate the importance of our theoretical analysis, which implies that there exists a gap between the theoretical prediction and numerical results about the minimum number of pinning controlled nodes.

1. Introduction

Over the past few decades, complex dynamical networks have attracted a great deal of attention from different fields of scientific research [1,2]. Examples of such complex networks can be found in various fields of real world, such as in the Internet, biological systems, scientific citation web, etc., and thus become an important part of our daily life. The ubiquity of complex networks has naturally resulted in a series of important research problems concerning how the network structure facilitates and constrains the network dynamical behaviors. In particular, special attention has been focused on synchronization and control problems of complex dynamical networks.

Synchronization is a phenomenon which is of fundamental scientific interest in linear or nonlinear complex dynamical systems [3]. In particular, complete synchronization, characterized by all nodes converging to a uniform dynamical behavior in complex networks, has become a focal research topic recently. As the complex network cannot synchronize by itself, there occur many control techniques to drive the network to synchronize. However, in the real world, complex networks have a large number of nodes, and it is usually impractical to control a complex network by adding the controllers to all nodes. To reduce the number of controlled nodes, pinning control, in which controllers are only applied to partial nodes, is introduced. There are some novel synchronization criteria in complex networks with delay or non-delay coupling. We refer the readers to Guo et al. [4] and Liu et al. [5] for pinning synchronization of delayed dynamical networks with linear coupling and Xia and Cao [6] by periodically intermittent control. Moreover, in Ref. [7], the authors addressed the problem of pinning synchronization based on the more detailed structural information of general complex dynamical networks, particularly strongly connected networks, networks with a directed spanning tree, weakly connected networks, and directed forests, etc. It was also pointed out that it is still quite challenging to consider the pinning scheme on a general complex network. In Ref. [8], a try has been made to determine the minimum number of nodes needed to be pinned to reach network synchronization.

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It is known that, in many practical problems, it often happens that the coupling scheme is nonlinear [9–11]. Liu and Chen [9] discussed the synchronization of dynamical networks with an asymmetrical coupling matrix. Mahdavi, Menhaj and Kurths [11] studied pinning impulsive synchronization of complex dynamical networks with nonlinear coupling. But for a nonlinear coupled delay complex dynamical network, it is still an interesting but very difficult task to explore the effect of nonlinear coupling strengthen and delay on synchronization by pinning control. Although some delayed models have been discussed by Zhou et al. in Ref. [12], we find that, when choosing $g_1(x) = x + \sin(x)$, it is impossible to prove the validity of Hypothesis 4′ [12] as

$$(g_1(x) - g_1(y))/(x - y) \geq d_1 > 0 \quad (x, y \in R)$$

in a mathematical way, where d_1 is a positive constant. This case will be discussed as an example in the numerical simulation part (please see Section 4) of this paper. Moreover, it is shown that complex networks such as the Watts–Strogatz (WS) random [2] or scale-free BA models [1] can be synchronized by pin-controlling fewer nodes than regular systems. A gap cannot be filled between the theoretical prediction and numerical simulation, which needs a further study in the future. These motivate us to write this paper.

The rest of the paper is organized as follows: in Section 2, we propose a general complex dynamical network model; some preliminaries and lemmas are included. In Section 3, some pinning synchronization criteria for the general complex delayed dynamical networks with nonlinear coupling are given. Numerical examples are given in Section 4. Finally, we draw our conclusion in Section 5.

2. Preliminaries and mathematical models

Consider a generally controlled complex delayed dynamical system consisting of N nodes, with each node being of m dimensions, which can be described as follows:

$$\dot{x}_i(t) = Ax_i + f(x_i(t)) + c\sum_{j=1}^{N} b_{ij}\Gamma x_j(t) + c_1\sum_{j=1}^{N} \bar{b}_{ij}\Gamma g(x_j(t-\tau)) + v_i(t), \tag{2.1}$$

where $1 \leq i \leq N$, $x_i(t) = (x_{i1}(t), \ldots, x_{im}(t))^T \in \mathbb{R}^m$ is the state variable of node i, $A \in \mathbb{R}^{m\times m}$ is a given constant matrix, and $f : \mathbb{R}^m \to \mathbb{R}^m$ is a continuously differentiable function describing the nonlinear dynamics of the single node. Here, c and c_1 are two parameters of the non-delay and delay coupling strengthen, respectively, $\Gamma = (\gamma_{ij}) \in \mathbb{R}^{m\times m}$ is an inner-coupling matrix, $\tau > 0$ is the coupled delay, and $v_i(t)$ is an adaptive controller. $B = (b_{ij})_{N\times N}$ and $\bar{B} = (\bar{b}_{ij})_{N\times N}$ represent the adjacency configuration of the network with the non-delay and delay couplings, respectively, which can be described as a random network by the E-R model [1], a scale-free random network proposed in Ref. [1] (BA models), a small-world random network proposed by Watts–Strogatz [2] or a regular network respectively. If there is a link from node i to node j, then $b_{ij} > 0$ (or $\bar{b}_{ij} > 0$) ($j \neq i$); otherwise, $b_{ij} = 0$ (or $\bar{b}_{ij} = 0$). Moreover, $b_{ii} = -\sum_{j\neq i} b_{ij}$ and $\bar{b}_{ii} = -\sum_{j\neq i} \bar{b}_{ij}$.

Note that the inner-coupling matrix Γ and coupling configuration matrix B or $\bar{B}$ do not need to be symmetric, and the corresponding topological graph can be directed or undirected. Throughout the paper, we always assume that $g(x(t))$ satisfies the uniform Lipschitz condition with respect to the time t,

$$\|g(x(t)) - g(y(t))\|_2 \leq L\|x(t) - y(t)\|_2.$$

Hereafter, let $s(t) = s(t; t_0, x_0)$ $(\in \mathbb{R}^m)$ be a solution of the single node system

$$\dot{x} = Ax + f(x).$$

Then $s(t)$ is a synchronous state of the controlled complex delayed dynamical system (2.1). Note that $s(t)$ may be an equilibrium point, a periodic orbit, or even a chaotic attractor.

Definition 1. If $\lim_{t\to\infty} \|x_i(t) - s(t)\|_2 = 0$, $(i = 1, 2, \ldots, N)$, then the controlled complex delayed dynamical system (2.1) is said to achieve synchronization.

Define error vectors as

$$e_i(t) = x_i(t) - s(t), \quad 1 \leq i \leq N.$$

According to the system (2.1), the error system is described by

$$\dot{e}_i(t) = Ae_i + f(x_i(t)) - f(s(t)) + c\sum_{j=1}^{N} b_{ij}\Gamma e_j(t) + c_1\sum_{j=1}^{N} b_{ij}\Gamma(g(x_j(t-\tau)) - g(s(t-\tau))) + v_i(t), \tag{2.2}$$

where $1 \leq i \leq N$.