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Detrending moving-average cross-correlation coefficient: Measuring cross-correlations between non-stationary series

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HIGHLIGHTS

- DMCA coefficient is introduced.
- Different settings (non-stationarity level, scales, correlations, time series length) are examined.
- DMCA coefficient is both an alternative and a complement to the DCCA coefficient.

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ABSTRACT

In the paper, we introduce a new measure of correlation between possibly non-stationary series. As the measure is based on the detrending moving-average cross-correlation analysis (DMCA), we label it as the DMCA coefficient $\rho_{DMCA}(\lambda)$ with a moving average window length λ. We analytically show that the coefficient ranges between -1 and 1 as a standard correlation does. In the simulation study, we show that the values of $\rho_{DMCA}(\lambda)$ very well correspond to the true correlation between the analyzed series regardless the (non-)stationarity level. Dependence of the newly proposed measure on other parameters – correlation level, moving average window length and time series length – is discussed as well.

1. Introduction

Inspection of statistical properties of multivariate series has become a topic of increasing importance in econophysics. For this purpose, various estimators of power-laws in cross-correlations of a pair of series have been proposed—detrended cross-correlation analysis and its various versions [1,2], detrending cross-correlation moving average [3,4], height cross-correlation analysis [5] and cross-correlation analysis based on statistical moments [6]. Out of these, the detrended cross-correlation analysis (DCCA) [1,2] has become the most popular one. Apart from the analysis of the power laws in the cross-correlation function itself, Zebende [7] proposed the DCCA cross-correlation coefficient as a combination of DCCA and the detrended fluctuation analysis (DFA) [8–10]. Even though its ability to uncover power-law cross-correlations has been somewhat disputed [11–14], Kristoufek [15] shows that the coefficient is able to estimate the correlation coefficient between non-stationary series precisely and that it dominates the standardly used Pearson's correlation coefficient.

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Historically, the detrended fluctuation analysis and methods derived from it have been frequently compared to (or sometimes even interwoven with) the detrending moving average (DMA) [16,17] procedures. In most cases, the competing approaches fare very similarly while the DMA algorithms are computationally less demanding as they contain no box-splitting[1] and regression fitting [18–26]. In this paper, we follow these steps and propose an alternative but also a complementary coefficient to the DCCA cross-correlation coefficient of Zebende [7]—the detrending moving-average cross-correlation coefficient. In the following section, the coefficient is introduced. After that, results of the wide Monte Carlo study are presented showing that the newly proposed coefficient estimates the true correlation coefficient precisely even for strongly non-stationary series. Comparison to the DCCA coefficient is included as well.

2. DMCA coefficient

We start with the detrending moving average procedure (DMA) proposed by Vandewalle & Ausloos [16] and further developed by Alessio et al. [17]. For (possibly asymptotically non-stationary) series $\{x_t\}$ and $\{y_t\}$, we construct integrated series $X_t = \sum_{i=1}^{t} x_i$ and $Y_t = \sum_{i=1}^{t} y_i$ for $t = 1, 2, \ldots, T$ where T is the time series length which is common for both series. Fluctuation functions $F_{x,DMA}$ and $F_{y,DMA}$ are then defined as

$$F^2_{x,DMA}(\lambda) = \frac{1}{T-\lambda+1} \sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \left(X_t - \widetilde{X_{t,\lambda}}\right)^2, \tag{1}$$

$$F^2_{y,DMA}(\lambda) = \frac{1}{T-\lambda+1} \sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \left(Y_t - \widetilde{Y_{t,\lambda}}\right)^2 \tag{2}$$

where λ is the moving average window length and θ is a factor of moving average type (forward, centered and backward for $\theta = 0, \theta = 0.5$ and $\theta = 1$, respectively). $\widetilde{X_{t,\lambda}}$ and $\widetilde{Y_{t,\lambda}}$ then represent the specific moving averages with the window size λ at time t. Different types of moving averages have been studied and the centered one ($\theta = 0.5$) shows the best results [18] so that we apply $\theta = 0.5$ in this study as well.

For the bivariate series, He & Chen [4] propose the detrending moving-average cross-correlation analysis (DMCA) which is a special case of the method proposed by Arianos & Carbone [3]. The bivariate fluctuation F^2_{DMCA}, which can be seen as a detrended covariance, is defined as

$$F^2_{DMCA}(\lambda) = \frac{1}{T-\lambda+1} \sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \left(X_t - \widetilde{X_{t,\lambda}}\right)\left(Y_t - \widetilde{Y_{t,\lambda}}\right). \tag{3}$$

In the steps of Zebende [7], we propose the detrending moving-average cross-correlation coefficient, or also the DMCA-based correlation coefficient, as

$$\rho_{DMCA}(\lambda) = \frac{F^2_{DMCA}(\lambda)}{F_{x,DMA}(\lambda)F_{y,DMA}(\lambda)}. \tag{4}$$

In a similar way as for the DCCA correlation coefficient [11], the DMCA coefficient can be rewritten as

$$\begin{aligned}\rho_{DMCA}(\lambda) &= \frac{F^2_{DMCA}(\lambda)}{F_{x,DMA}(\lambda)F_{y,DMA}(\lambda)} \\ &= \frac{\frac{1}{T-\lambda+1}\sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \left(X_t - \widetilde{X_{t,\lambda}}\right)\left(Y_t - \widetilde{Y_{t,\lambda}}\right)}{\sqrt{\frac{1}{T-\lambda+1}\sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \left(X_t - \widetilde{X_{t,\lambda}}\right)^2 \frac{1}{T-\lambda+1}\sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \left(Y_t - \widetilde{Y_{t,\lambda}}\right)^2}} \\ &= \frac{\sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \epsilon_{x,t}\epsilon_{y,t}}{\sqrt{\sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \epsilon^2_{x,t} \sum_{i=\lfloor \lambda-\theta(\lambda-1)\rfloor}^{\lfloor T-\theta(\lambda-1)\rfloor} \epsilon^2_{y,t}}}\end{aligned} \tag{5}$$

where $\{\epsilon_{x,t}\}$ and $\{\epsilon_{y,t}\}$ are the series $\{X_t\}$ and $\{Y_t\}$, respectively, detrended by the centered moving average of length λ. From the last part of Eq. (5), it is visible that

$$-1 \le \rho_{DMCA}(\lambda) \le 1 \tag{6}$$

1 Note that moving averages are also utilized in the detrended fluctuation analysis methods where the polynomial detrending is substituted by the moving average filtering [25]. However, DMA presented in this paper is based on scaling of fluctuations with moving average window length whereas DFA methods using moving averages are still based on box splitting and scaling with box sizes.