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Random walk–percolation-based modeling of two-phase flow in porous media: Breakthrough time and net to gross ratio estimation

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HIGHLIGHTS

- A random walk–percolation method is developed to model fluid flow in porous media.
- The occupancy probability significantly affects the breakthrough time distribution.
- The entropy plot is used to determine the net to gross ratio.

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ABSTRACT

Fluid flow modeling in porous media has many applications in waste treatment, hydrology and petroleum engineering. In any geological model, flow behavior is controlled by multiple properties. These properties must be known in advance of common flow simulations. When uncertainties are present, deterministic modeling often produces poor results.

Percolation and Random Walk (RW) methods have recently been used in flow modeling. Their stochastic basis is useful in dealing with uncertainty problems. They are also useful in finding the relationship between porous media descriptions and flow behavior.

This paper employs a simple methodology based on random walk and percolation techniques. The method is applied to a well-defined model reservoir in which the breakthrough time distributions are estimated. The results of this method and the conventional simulation are then compared. The effect of the net to gross ratio on the breakthrough time distribution is studied in terms of Shannon entropy. Use of the entropy plot allows one to assign the appropriate net to gross ratio to any porous medium.

1. Introduction

Simulation of multi-phase flow in porous media is of great importance in many disciplines, including contaminant hydrology and petroleum engineering. Modeling of multi-phase flow is difficult since the forces involved (e.g., viscous, capillary, gravity, and diffusion) may act at different characteristic scales. The complexity of porous media also makes things more challenging. In enhanced oil recovery (EOR) processes, flow simulation requires numerically solving the governing partial differential equations (PDEs). The popular finite difference method often leads to problems of stability, numerical

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dispersion, grid effect, and long computational time [1,2]. More advanced methods, such as finite elements and streamline simulation, have been developed to resolve these issues [3–8].

Percolation theory has been successfully applied in different scientific fields, including the spread of disease [9], forest fires [10], social models [11], polymer gelation [12] and porous media [13]. In percolation theory, the problem domain is discretized, then individual sites are declared to be either occupied or not. Neighboring occupied sites form clusters of different shapes and sizes. The mean cluster size depends on the occupancy probability (p), which is defined as the fraction of occupied sites to total sites. If the p value exceeds the percolation threshold (p_c), then the spanning cluster will connect the opposite sides of the system [13–19]. A few universal scaling laws describe the global behavior and properties of percolating systems [14,15]. Some applications of this theory in porous media include the determination of connectivity, effective permeability, breakthrough time and post breakthrough behavior [14,15,17,20–22].

The Random Walk (RW) method monitors the paths of many particles (walkers) as they randomly move in a domain. This helps us to understand how Brownian motion, diffusion, and fluid flow happen in a complex system. RW modeling has several advantages over the more common finite difference modeling: it can model the diffusion process and small scale heterogeneity. It also needs less computational time [23–25].

Lee et al. [26] studied the traveling time and length in a bond percolation domain using the Particle Launching Algorithm (PLA). The pressure values were found by Kirchhoff's law. They assumed that a tracer particle would select an outgoing bond with a probability proportional to the flow velocity. The scaling relations and distribution functions were also discussed. Araújo et al. [27] later used the same approach to investigate immiscible flow in the case of a long correlation length.

King et al. [22,28–31] applied the scaling laws to determine the breakthrough time of a passive tracer. They were able to regenerate the results of the conventional simulation. Andrade et al. [32] proposed a scaling law for the distribution function of the shortest path. Loṕez et al. [33] examined the post breakthrough behavior for both the homogeneous ($p = 1$) and heterogeneous cases ($p = p_c$). Araújo et al. [34] verified the statistics of the mass of invaded clusters between two sites. They showed that the mass followed a power law under certain conditions.

Andrade et al. [35] investigated how the viscosity contrast between displaced and injected fluids influenced the results. They found that the distributions of the breakthrough time and mass of the invaded clusters had similar scaling relations. However, in comparison with the case of unit viscosity ratio, the critical exponents were different. Oliveira et al. [36] studied the effect of a temperature gradient on oil recovery in a 2D pore-network model. They extended the scaling exponents of the breakthrough time distribution to cases different from the unit viscosity ratio [37].

In this work, some simple but realistic rules of fluid mechanics and probability theory are combined to model fluid flow. The resulting method, which is based on random walk modeling in percolating clusters, is applied to a standard reservoir [38]. The breakthrough time distributions are then compared with that of a commercial simulator. Finally, the Shannon entropy is used to determine a suitable value of the net to gross (NTG) ratio.

2. Description of random walk–percolation methodology for flow modeling

Some sections of porous media may take no part in fluid flow. In particular, the low permeability zones have little capacity to transmit fluids. If these zones are completely included in a flow simulation, the solution process will be time consuming. For all practical purposes, such regions may be excluded from flow modeling. In a simple method, those grid blocks with permeability less than some predefined cut off values (e.g., 1 md) become inactive. The main drawback of this method is that some high permeable regions may disconnect from each other. This can have dramatic effects on simulation results, such as the pressure field, flow paths and breakthrough time.

An advantage of the Random Walk–Percolation (RW–P) method over the more common finite difference simulation method is its stochastic basis. The method models numerous realizations of a porous medium. Each realization generates a spanning cluster that connects any pair of operating wells to each other. The concept of spanning cluster in underground flow applications is different from that of physics. In the latter, the spanning cluster develops when the two opposite sides of the system are connected [13,21,26,32]. A petroleum engineer, however, wants to monitor the flow from injection to production wells. Since the flow movement depends on the pressure distribution, it is crucial to include every active well. A realization that connects the two opposite sides, but leaves an active well out of the spanning cluster is not acceptable.

The traditional procedure for creating a realization begins by randomly selecting grid blocks until the occupancy probability is reached [26,27,32]. After using Hoshen and Kopelman's algorithm [39], the formation of the spanning cluster is checked. Cells that are not in the spanning cluster are impermeable (i.e., have zero permeability).

In the simulation, three types of blocks are recognized, namely the blocks within the spanning cluster (type 1), those that belong to other clusters (type 2) and, finally, unoccupied cells (type 3). Fig. 1 illustrates this concept in which *SC* and C#1 denote blocks of the spanning cluster and cluster#1, respectively. The number of blocks that are present in any cluster (the summation of first and second types) to the number of total cells is called the occupancy probability (p). For example, $p = (10 + 3 + 2)/30$ or $p = 0.5$ for the system shown in Fig. 1. The occupancy probability is usually interpreted as the Net To Gross (NTG) ratio in petroleum engineering. In this paper, these two expressions are used interchangeably.

Completely random selection may be questionable from the perspective of flow in porous media. Since higher permeability zones have a higher fluid flux, it is reasonable to link the chance of block selection to permeability. This means that the higher permeability cells have a greater chance of being selected. For example, a block with 10 md permeability will be