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Reaching consensus on rumors

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HIGHLIGHTS

- We provide an exact formalization of Galam diffusion of rumors model.
- The formalization shows the presence of impasses, which were overlooked in the previous literature.
- The proposed formulation allows a deeper and more comprehensive analysis of the diffusion of rumors.

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ABSTRACT

An important contribution in sociophysics is the Galam's model of rumors spreading. This model provides an explanation of rumors spreading in a population and explains some interesting social phenomena such as the diffusion of hoaxes. In this paper the model has been reformulated as a Markov process highlighting the stochastic nature of the phenomena. This formalization allows us to derive conditions for consensus to be reached and for the existence of some interesting phenomena such as the emergence of impasses. The proposed formulation allows a deeper and more comprehensive analysis of the diffusion of rumors.

1. Introduction

Consensus in groups has attracted interest from different disciplines, namely social psychology, economics, sociology and political sciences. For example, the influence of social pressure exercised by a minority is studied in Ref. [1] where the authors analyze how behavioral style may be a general source of influence. The process by which a group reaches consensus has been formalized and mathematically analyzed in Refs. [2,3] providing also simple conditions determining whether it is possible for the group to reach consensus. Another important contribution is provided by [4] which introduces a time-changing influential matrix and provides a sufficient condition to obtain convergence on the influential weights. More recent contributions based on the same modeling framework account for *strategic interaction* and study the existence of possible consensus equilibria when strategic interaction and social influence are combined together. For example, [5] incorporates in a DeGroot-like model, see again [2], the possibility that agents misrepresent their opinion with the intent of reaching a conformity. Indeed, adopting a behavior that is different from the others might cause disutility. In this case, the combination of this strategic acting and the social influence requires additional conditions to reach consensus. Following a similar idea, [6] introduces an extension of the classical DeGroot model of opinion formation for studying the transmission of cultural traits in an overlapping generation setting, where parents strategically display a cultural trait to influence their children.

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Sociophysics as well, has devoted a lot of attention to social influence, starting from the pioneering works by Galam (see, e.g., Ref. [7]) to recent contributions, such as [8]; for a review see Refs. [9,10]. In his contributions [11,12], Galam moves the perspective from within the group to a whole population whose individuals try to choose an opinion (true or false) on a rumor on the basis of repeated discussions in social gatherings. At each of them, a small group of people get together and line up with a consensual opinion in which everyone agrees with the majority inside the group. His model provides a plausible explanation for the diffusion or self-propagation of rumors through free public debates, such as the temporary diffusion of the so called Pentagon French hoax, according to which no plane crashed on the Pentagon on September the 11th. Indeed, although initially supported only by a minority of the population, this rumor started to propagate with an astonishing and unexpected adhesion till the moment in which a strong media campaign carried out by newspapers reversed the process.

In the Galam's formulation of the model the binomial distribution has been used to approximate the probability of having a specific table seating configuration. The use of the binomial distribution is a good modeling approximation as long as populations with a large number of agents and small size discussion group are considered. Nevertheless, this approach may create distortions in the prediction of the dynamics of rumors in small communities. In order to deal with this problem, we formalize the process as an absorbing Markov chain, in which the states of the process correspond to the number of agents holding one opinion. Our approach extends Galam's analysis providing further results and precision. The goal is to have further information about the process of diffusion of opinions which are not possible to obtain using the original Galam's formulation. In particular, we aim to investigate thoroughly the evolution of the rumors spreading process identifying all the absorbing states and computing the probability to reach each of them.

The structure of the paper is the following: In Section 2 the model is described as an absorbing Markov chain. In Section 3 the stochastic killing point is defined. This is the stochastic version of the killing point introduced in Ref. [12]. In Section 4 the formula for the conditional expected values is provided. Finally, in Section 5 further possible developments of the model are suggested.

2. Formalization of the process

Consider a N person finite population and assume that only two opinions, '$+$' and '$-$', are possible. Assume that at time $t = 0, 1, \ldots$ each individual holds either one or the other opinion and Y^t denotes the number of those holding opinion '$+$' at time t. The set of possible states of the population with respect to opinion '$+$' is therefore $S = \{0, 1, \ldots, N\}$, where state 0 means consensus has been reached on opinion '$-$' while, on the contrary, state N means consensus has been reached on opinion '$+$'. Not all the states in S are necessarily feasible. State feasibility depends on aspects, which will be introduced later, such as *social space* and *discussion functions*.

As in Refs. [12–14] the interaction takes place at different size tables. The *social space*, where the discussion takes place, is the set of tables $\mathcal{N} = \{T_1, T_2, \ldots, T_L\}$ with[1] $L < N$. Let $|T_r|$ be the size of table T_r with $\sum_{T_r \in \mathcal{N}} |T_r| = N$. The table sizes, i.e., the number of people that can be seated at a given table, can be summarized in vector $\mathbf{n} = (|T_1|, |T_2|, \ldots, |T_L|) \in \mathbb{R}^L$. As the number of seats at each table is the only relevant variable, the social space can be denoted either by $\mathcal{N}$ or $\mathbf{n}$; furthermore we assume as in Ref. [12] that the social space remains the same during the whole process.

Given a social space $\mathcal{N}$, we can determine how may tables have size k as follows

$$\tau_k = \sum_{T_r \in \mathcal{N}} \delta_{k,|T_r|} \tag{1}$$

where $\delta_{k,|T_r|}$ is the Kronecker's delta.

As we assume the social space being fixed over time, the probability a_k to be seated at a size k table is stationary and can be determined as follows

$$a_k = \frac{k}{N}\tau_k \quad \text{with } k = 1, \ldots, K, \tag{2}$$

where K is the number of seats of the largest size table in the social space. Given the social space $\mathcal{N} = \{T_1, T_2, \ldots, T_L\}$, vector $\mathbf{y} = (y_1, y_2, \ldots, y_L)$ indicates a generic seating configuration where y_1 agents with opinion '$+$' are seated at table T_1, y_2 are seated at table T_2,... and y_L are seated at table T_L, with the obvious feasibility conditions $0 \le y_r \le |T_r|, r = 1, 2, \ldots, L$. Therefore entries in $\mathbf{y}$ depend[2] on the social space.

Assuming $y \in S$ agents with opinion '$+$', let us introduce the set Ω_y as

$$\Omega_y = \left\{ \mathbf{y} : 0 \le y_r \le \min(|T_r|, y), r = 1, 2, \ldots, L \text{ and } \sum_{r=1}^{L} y_r = y \right\}.$$

This set consists of all possible seating configurations of y agents with opinion '$+$' given the social space.

The probability of each seating configuration $\mathbf{y} \in \Omega_y, \forall y \in S$ can be computed as follows

1 The case $L = N$, i.e. all table of dimension 1, is trivial and therefore will not be considered.

2 For the sake of simplicity we can avoid using the heavier notation $\mathbf{y_n}$ as the social space is assume fixed.