Physica A 404 (2014) 158–170

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Localization properties of one-dimensional speckle potentials in a box

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HIGHLIGHTS

- We study the box-confined single particle with 1D speckle disorder.
- Finite 1D systems can show Anderson localization without effective mobility edge (EME).
- We find and explain uncommon degeneracy phenomena amongst low energy eigenstates.
- When lacking EME, there is a different delocalization process when increasing energy.

ARTICLE INFO

Article history:
Received 22 September 2013
Received in revised form 20 January 2014
Available online 28 February 2014

Keywords:
Speckles
Anderson localization in disordered solids
Mobility edges

ABSTRACT

We investigate the localization properties of the single particle spectrum of a one-dimensional speckle potential in a box. We consider both the repulsive and the attractive cases. The system is controlled by two parameters: the size of the box and a rescaled potential intensity. The latter is a function of the particle mass, the correlation length and the average intensity of the field. Depending on both these parameters values and the considered energy level, the eigenstates exhibit different regimes of localization. In order to identify the regimes for the excited states, we use a technique developed in this work. Depending on the chosen parameters values, we find that it is possible not observing any effective mobility edge nor delocalization of the eigenstates due to the finite size of the system.

1. Introduction

Random potentials created by laser speckles are routinely employed in experiments with ultracold atoms to investigate the behaviour of disordered systems [1–4]. One-dimensional (1D) speckle potentials in particular have been the object of an intensive study in recent years. From both the experimental and the theoretical side, many interesting features have been addressed. These include classical localization and fragmentation effects, frequency shifts and damping of collective excitations, inhibition of transport properties, Anderson localization and related phenomena [5–15].

The theoretical investigation on the 1D speckle potential and, more in general, on the 1D disordered systems focuses on the infinitely extended case and nowadays a clear framework has been outlined. In 1D disordered systems, all states are exponentially (aka Anderson) localized, regardless to the weakness of the random potential [16]. The quantity that is commonly used to describe this phenomenon is the eigenstates inverse localization length (or Lyapunov exponent), which has been shown to be strongly dependent on the shape of the considered potential. In the Born approximation regime, the Lyapunov exponent as a function of energy results to be simply proportional to the Fourier transform of the potential auto-correlation function [12,17]. Therefore, if the power spectrum of a given disordered potential has a finite support (i.e. if the potential self-correlation is long ranged), Lyapunov exponent goes to zero beyond the critical energy E_C associated with the

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http://dx.doi.org/10.1016/j.physa.2014.02.064
0378-4371/

highest available Fourier component. This fact defines the so called effective mobility edge. The incongruity between the prediction above and the presence of localization at every energy level can be solved by considering the perturbative expansion of the Lyapunov exponent beyond the Born approximation (in Ref. [14] this is proven for the 1D speckle potentials). In fact, the localization length remains finite also beyond E_C, due to the contribution of the $n > 2$ sites correlators, though showing a kink while crossing the energy threshold. Since all the eigenstates are localized, the effective mobility edge is not, in any sense, an Anderson transition and so the 1D disordered systems do not exhibit any multifractal behaviour at E_C [18].

Other interesting results have been obtained for the 1D speckle potentials. Regarding the uniform case, the low energy behaviour has been first considered in Ref. [10], where it has been shown that density of states in a repulsive speckle potential is characterized by a Lifshitz tail [19,20]. As for both attractive and repulsive speckles, this has been discussed more thoroughly in Ref. [21], where it has also been shown that three different regimes of speckle intensities s can been identified at low energy—semiclassical ($s \gg 1$), intermediate, and quantum ($|s| \leq 1$). The presence of an effective mobility edge have been observed in the Anderson localization experiment [13]. Furthermore, the existence of both *extended* (compared with the finite length scale of the system) and localized states in the presence of an inhomogeneous confinement has been discussed in Ref. [22].

Despite this intense research activity, the single particle spectrum properties for a speckle potential have been only partially addressed, even in the 1D case. In fact, the framework summarized above has been developed considering only infinitely extended systems. The finite sized system is usually employed just as a numerical tool to verify the theoretical predictions, provided that it is large enough to be a good approximation of the infinitely extended case. What is lacking so far is a detailed numerical analysis of how the eigenstates density profile of the finite sized speckle system is related to the main properties of the potential depending on the chosen energy level (i.e.: spatial extension, intensity, blue or red detuning). More specifically, we want to investigate the 1D speckle systems with an intermediate size between the *large* ones, reproducing the infinitely extended case behaviour, and the *short* ones, where the localization length is greater than the size and so the eigenstates are extended. For these systems the correlators cannot be properly defined beyond the system length scale and a similar consideration holds for the definition of the Lyapunov exponent. To the best of our knowledge, there is not a clear picture that describes to what extent the finite size affects predicted behaviours such as the effective mobility edge.

In this article we consider a 1D speckle potential delimited by infinite walls – a sort of *disorder in a box* – and we discuss the localization properties of its eigenstates as the speckle intensity s and the size of the system L changes. As discussed in Ref. [23], the box-like barriers can be achieved experimentally by using two laser beams that propagate perpendicular to the speckle direction, allowing for the investigation of finite size effects in a textbook case.

We study both the ground state and the excited states, with the aim of characterizing the system localization properties while varying the parameters s (amplitude of the potential), L (size of the system) and the energy level E. When possible, we extrapolate the system behaviour in the limit $L \to \infty$.

We observe that the centre of mass of some of the eigenstates shifts stochastically while varying s, this depending on the single realization of the speckle potential. A numerical measure of the probability of observing this phenomenon is compared with a theoretical framework developed in this work. We are then able to separate the contribution due to the boundary effects from the one depending only on the shape of the disordered potential. We express the latter as a function of the participation ratio.

We address the role played by the finite size of the system L and we compare our numerical results with the theory. Properly choosing L and s, we observe a regime which is different from the predicted one for $L \to \infty$. In this regime, eigenstates are still localized at $E \geq E_C$ (L is big enough to make the boundaries role not to be predominant). However, the effective mobility edge is completely absent. As L gets bigger, we observe the approach of the system to the predicted behaviour.

Although the considered system is not able to show a true multifractal behaviour, due to the absence of any Anderson transition, we use some traditional tools of multifractal analysis to define a functional able to distinguish localized eigenstates from extended ones and from the ones belonging to the crossover region of the spectrum. We verify that this observable is more effective than traditional ones and so we use it to characterize the three regions of the spectrum while varying s.

The article is organized as follows. In Section 2 we define both the system we consider and the main tools we use to characterize its eigenstates. In Section 3 we discuss the properties of the ground state and how its appearance depends on s and L. The density profile of the excited states is considered in Section 4. Our results are summarized in Section 5.

2. Model and methods

Let us consider a single particle in a 1D speckle potential $V_s(x) = V_0 v(x/\xi)$, with intensity $V_0 = \langle V_s \rangle$ and autocorrelation length ξ [24,8]. The probability distribution of $v(x)$ is e^{-v}. Moreover it holds that

$$\langle v(x)v(x+\Delta)\rangle = 1 + \mathrm{sinc}^2\left(\frac{\Delta}{\xi}\right).$$

Optical speckle is obtained by transmission of a laser beam through a medium with a random phase profile, such as a ground glass plate. The resulting complex electric field is a sum of independent random variables and forms a Gaussian process. Atoms experience a random potential proportional to the intensity of the field. V_0 can be both positive or negative, the potential resulting in a series of barriers or wells, that in the following will be referred as *repulsive* (or blue) and *attractive* (or red), respectively. The different features of the system in the red and the blue cases arise from the different distributions