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Percolation of interacting classical dimers on the square lattice

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HIGHLIGHTS

- Ising clusters are defined based on the interacting dimer configurations.
- As temperature increases, the model undergoes a percolation transition.
- The percolation point is $T_c = 0.654(2)$, coinciding with the model's KT critical point.
- The largest cluster at $T_c = 0.654(2)$ is a fractal with dimension $D_c = 1.874(2)$.
- D_c coincides with the value of exponent of the dimer–dimer correlation function.

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ABSTRACT

We study the percolation properties of the interacting classical dimer model on the square lattice by means of Monte Carlo simulations and finite-size scaling analysis. We define Ising clusters based on the dimer configuration; the percolation point of the clusters coincides with the critical point of the Kosterlitz–Thouless transition of the dimer model, which is $T_c = 0.654(2)$. Furthermore, we find that the largest cluster at the Kosterlitz–Thouless point is a fractal, with fractal dimension $D_c = 1.874(2)$, which coincides with the critical exponent describing the critical behavior of the dimer–dimer correlation function, which is theoretically predicted to be 15/8.

1. Introduction

The problem of lattice coverings by hard objects, dimers in particular, has been studied for more than 70 years [1]. The combinatorial problem of finding the exact number of dimer coverings for planar lattice has been solved in the early 1960s by means of Pfaffian techniques [2–4], which have been extended to calculate dimer–dimer and monomer–monomer correlation functions [5]. Kasteleyn pointed out that the fully covered dimer model on the square lattice has no phase transition [6]. However, when the dimers on the square lattice have aligning interactions, the system undergoes a Kosterlitz–Thouless (KT) transition [7], which has been further confirmed by both Monte Carlo simulations and transfer matrix calculations [8].

In the current paper, we also study the interacting dimer model by means of Monte Carlo simulations. Instead of the thermodynamic properties of the model, we mainly pay attention to the percolation [9] properties of the model.

In statistical physics, the study of lattice models in terms of percolation is a hot topic. For example, the q-state Potts model [10,11] can be mapped to the random-cluster model [12,13], which can be considered as a generalized percolation

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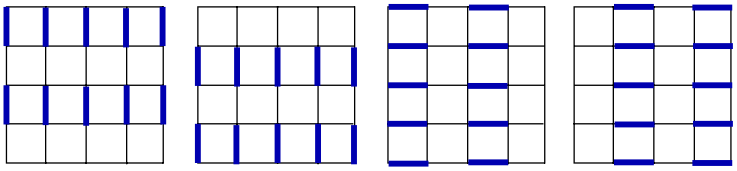

Fig. 1. Four ground states of the interacting dimer system.

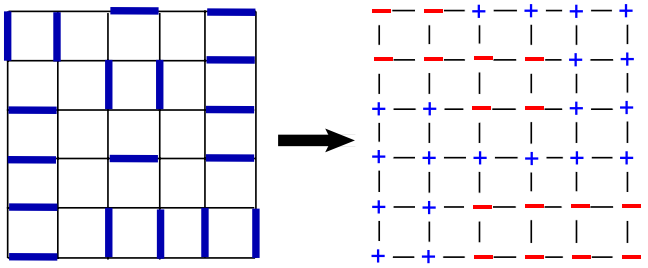

Fig. 2. A typical dimer configuration (left) and the Ising clusters defined on it (right).

model. When $q = 1$, it reduces to the simple bond percolation model [9]. For the interacting dimer model, it can also be mapped to a correlated percolation model by proper rules. We will show that the percolation point coincides with the critical point of the KT transition of the model and the critical configuration is a fractal.

The paper is arranged as follows: in Section 2 we introduce the interacting dimer model and define Ising clusters based on the dimer configuration. In Section 3 we describe the algorithm we used and the variables we sampled in our Monte Carlo simulations. In Section 4 we give our numerical results. At last, we conclude our paper in Section 5.

2. The model and the definition of clusters

The interacting dimer model on the square lattice is defined by the partition function [7]

$$Z = \sum_c \exp\left(-\frac{E_c}{T}\right), \tag{1}$$

where T is the temperature and $E_c = \nu\,[N^c(=) + N^c(\|)]$ is the energy. c is the dimer covering configuration and $N^c(=) + N^c(\|)$ is the number of plaquettes with parallel dimers. As in Ref. [7], we set $\nu = -1$, thus the dimers have aligning interactions and the system favors configurations with parallel occupation of dimers. The ground state of the system is fourfold degenerate, as shown in Fig. 1. The phase transition, which is of KT type [7,8], is between this columnar phase (at low temperature) and the high-temperature phase.

To study the percolation properties of the dimer model as that in Ising model [14,15], we define the Ising clusters based on the dimer configuration. This is completed by simple rules:

1. *Ising spins*: assume that there is an Ising spin on each site.
2. *Ising configuration*: as shown in Fig. 2, for each site, if it is covered by a horizontal dimer, set the sign of the spin to be positive; if it is covered by a vertical dimer, set the sign of the spin to be negative.
3. *Ising cluster*: if two nearest-neighbor Ising spins have the same sign, they are considered to be in the same cluster.

Alternatively, in the definition, one can also let the sign of spins covered by horizontal dimers be negative and the sign of spins covered by vertical dimers be positive. Each dimer configuration corresponds to an Ising configuration which consists of Ising clusters. It is obvious that the Ising configurations have spin-up–spin-down symmetry. It should be noted that the definition is only a technical trick for studying the model in terms of percolation, we have not mapped the dimer model to an effective Ising model; the simulations are done using the Hamiltonian of the interacting dimer model but not the Ising model.

After the clusters are defined, we can investigate the interacting dimer model in terms of percolation. Whether the percolation threshold of the Ising clusters coincides with the KT point of the model? Is the percolating cluster a fractal at the KT point? What is the fractal dimension of the percolating cluster? To answer these questions is our motivation for the present paper.

3. Algorithm and sampled variables

For simulating the interacting dimer model, the algorithm with local updates, such as the Metropolis algorithm or heat-bath algorithm, is not applicable. Because each site is occupied by one and only one dimer, adding or removing a dimer will