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Non-universality of the absorbing-state phase-transition in a linear chain with power-law diluted long-range connections

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HIGHLIGHTS

- The contact process on chains with power-law distributed connections is investigated.
- The absorbing-state phase transition deviates from the directed percolation universality class.
- We find a crossover from the 1D directed percolation to mean-field-like exponents when the couplings become longer ranged.

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ABSTRACT

In this work we study the critical behavior of the absorbing state phase transition exhibited by the contact process in a linear chain with power-law diluted long-range connections. Each pair of sites is connected with a probability $P(r)$ that decays with the distance between the sites r as $1/r^{\alpha}$. The model allows for a continuous tuning between a standard one-dimensional chain with only nearest neighbor couplings ($\alpha \to \infty$) to a fully connected network ($\alpha = 0$). We develop a finite-size scaling analysis to obtain the critical point and a set of dynamical and stationary critical exponents for distinct values of the decay exponent $\alpha > 2$ corresponding to finite average bond lengths and low average site connectivity. Data for the order parameter collapse over a universal curve when plotted after a proper rescaling of parameters. We show further that the critical exponents depend on α in the regime of diverging bond-length fluctuations ($\alpha < 3$).

1. Introduction

The Contact Process (CP), introduced by Harris a long time ago [1], was one of the first models to exhibit a non trivial critical behavior even in one dimension. Originally proposed to describe epidemic spreading, it has been later used to model a variety of competitive dynamical phenomena [2].

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The CP state is defined on a d-dimensional network, where each site is occupied by an active (infected) or an inactive (healthy) individual. Furthermore, the CP dynamics is a stochastic process obeying a Markovian rule: in a time step, healthy individuals may be infected with a probability p which depends both on the number of active as well as inactive neighboring individuals, while infected individuals may recover at constant probability λ, which is usually considered as a control parameter.

This rule generates a dynamical competition between infection and healing processes, whose output is determined by the value of the control parameter λ. It turns out that the fraction of infected individuals ρ (order parameter) vanishes for large values of λ while, for small values of λ, the ratio ρ remains strictly positive. Therefore, for large values of λ the epidemic is completely eliminated and the population reaches an absorbing healthy state. For small values of λ the system evolves towards a statistically stationary dynamical epidemic configuration with a fluctuating fraction of active individuals. The absorbing state will eventually be reached in systems with a finite size irrespective to the value of λ because random fluctuations will occasionally drive the system towards the healthy vacuum state.

According to the above characteristics, the system then displays a continuous transition from an active state to an absorbing state at a critical transition point λ_c. Many previous works have shown that the CP model belongs to the universality class of directed percolation (DP) [2]. These studies are in agreement with the conjecture of Grassberger and Janssen [3,4] that CP stays in the DP universality class since it only includes short-range interactions and does not have special attributes such as additional symmetries or quenched disorder [5]. All systems that belong to the DP universality class have a continuous phase transition to an absorbing state which is characterized by a strictly positive order parameter.

In recent years, many generalizations of the CP model have been proposed [6–12]. In particular, there has been an increasing interest in model systems with long-range interactions whose critical behavior can depart from the usual DP universality class [13–15]. The most straightforward proposal considered fat-tailed probabilities for the connections, which typically can be Levy or power-law type [16]. Another class of contact processes whose universality class deviates from DP includes models with an explicit particle diffusion process [17,18]. In these systems, the particles can have two distinct states, healthy or infected, and can independently diffuse. The total density of particles acts as the control parameter with the active state being statistically stable only above a critical concentration of particles. Even local diffusion is a relevant ingredient in these models and the critical properties of the absorbing state phase transition depends on the relative diffusivity of particles in distinct states. Recently, a superdiffusive epidemic process model was introduced whose critical exponents were shown to depend on the decay exponent of a Levy-like jump distribution function, exhibiting a crossover from the diffusive epidemic universality class to the mean-field universality class as the jump distribution becomes more long-ranged [19].

In this paper, we will advance along this research line, considering a Generalized Contact Process (GCP) model in one-dimension with strictly power-law distributed connections and no particle diffusion [20–22]. More precisely, the probability of a link between the sites i and j, namely $P(r_{ij} = |i-j|)$, decays as a power law (see Eq. (1) below). In this sense, the present model differs from those considering the activation process to be governed by interactions with other active individuals, even when they are located at long distances. It has been suggested that the contact process with Levy exchanges belongs to the same universality class of the contact process in models with long-distance interactions [21]. In this class of models, the infection probability decays as a power-law of the distance to the active individuals. Both variants of these models (interaction with all active individuals or just with the nearest one) deviate from the usual DP universality class, exhibiting continuously varying critical exponents [16,22,23].

The present model considers the contact process on a network on which all first-neighbors connections are included together with additional diluted long-distance connections, whose fraction decays as a power-law of the distance. The contact process on complex networks has also been a subject of current interest regarding its critical behavior. In scale-free networks, it has been suggested that the transition should fall in a generalized DP mean-field universality class, although numerical simulations have shown some deviations [24–28]. The contact process on a multi-scale network consisting of linear chains connected by a scale-free one also was shown to belong to a new universality class [29]. More recently, the contact process on chains with quasi-periodic connections has been investigated, and a new set of critical exponents were unveiled with a dependence on the underlying inflation rule [30]. Here, the exponent α governing the decay of the connection probability allows us a continuous variation from the one-dimensional contact process model with just first neighbors couplings ($\alpha \to \infty$), to the mean-field regime consisting in the fully connected network ($\alpha = 0$). Indeed, as we will see later, the parameter α shapes the nature of the interaction, and only for sufficiently large values of α (short-range contacts favored) the model falls in the standard *DP* universality class.

This paper is organized as follows: in Section 2 we describe our short/long range GCP model including the numerical techniques used. Section 3 deals with our results concerning the critical points, emphasizing their main features. The summary and conclusions of this paper are presented in Section 4.

2. Model and numerical methods

We define our GCP model taking into account a linear chain of length L with periodic boundary conditions. Two sites i and j of the chain are connected with a probability $P(r_{ij})$, independent of other pairwise connections, where r_{ij} denotes the smallest distance between sites along the closed chain. This probability is assumed to obey a power-law decay, i.e.

$$P(r_{ij}) = 1/r_{ij}^{\alpha} \tag{1}$$