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Time varying moments, regime switch, and crisis warning: The birth–death process with changing transition probability

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HIGHLIGHTS

- Calm and turbulent market can be revealed by high moment empirical representation.
- Regime switch can be shown by time-dependent transition probability.
- Price fluctuations by social interactions can be described by birth–death process.
- Rising 3rd to 5th moments one-quarter before the crisis provide warning signals.
- The master equation approach indicates conditions of market breakdown.

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ABSTRACT

The sub-prime crisis in the U.S. reveals the limitation of diversification strategy based on mean–variance analysis. A regime switch and a turning point can be observed using a high moment representation and time-dependent transition probability. Up–down price movements are induced by interactions among agents, which can be described by the birth–death (BD) process. Financial instability is visible by dramatically increasing 3rd to 5th moments one-quarter before and during the crisis. The sudden rising high moments provide effective warning signals of a regime-switch or a coming crisis. The critical condition of a market breakdown can be identified from nonlinear stochastic dynamics. The master equation approach of population dynamics provides a unified theory of a calm and turbulent market.

1. Introduction

Current economic literature has no consensus on the pertinent representation of financial crises. A neo-classical perspective, such as the Diamond–Dybvig model [1] and the noise trader model [2], only gives qualitative descriptions of multiple equilibriums, but they do not offer any operational indicator in defining different regimes in terms of empirical observation. The evolutionary perspective is interested in time patterns of historical events. Minsky and Kindleberger made a stylized description of three types of crises [3]. They made a verbal description of duration and phases in historical records, but did not suggest any quantitative measurement of a crisis in terms of a time series. These two perspectives are intuitive in theoretical ideas but impractical in quantitative analysis. How to diagnose a coming crisis from empirical data is an open issue both in theory and practice.

To bridge the gap between qualitative theory in a historical perspective and quantitative measurement in numerical experiments, two quantitative approaches are used for diagnosing a crisis in a time series analysis.

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The first approach tries to identify some thresholds for market bubbles from an excessive volatility of market indicators [4]. However, the following experiment demonstrates that price level changes are not a reliable indicator for a crisis. Based on historical records, we may rank a one-day price drop from high to low. The largest one-day drop of 22.61% occurred on Oct. 16, 1987, which did not develop into a significant crisis. The 2nd largest one-day price drop was 13.47% on Oct. 25, 1929. Among the top 30 events with one-day price drops larger than 6.54%, 17 events (57% of the top 30 events) occurred during the Great Depression. 4 events (13%) happened during the 2008 Crisis. 9 events (30% of total observations) did not trigger a full-fledged crisis. Their price drops ranged from 6.54% on Sept. 23, 1955 to 22.61% on Oct. 16, 1987. Clearly, judging a crisis based on the magnitude of level changes would be quite subjective, because there is no theory revealing the relationship between crises and price change magnitudes.

The second approach is based on some ad hoc static models, such as a fat tail distribution or a log-periodic model. Its strength is its mathematical simplicity, but its weakness is that it is hard to put its findings in a historical perspective. It is widely believed that high-frequency financial data have the accumulated probability distribution function (pdf), which decays with an inverse cube [5]. Preis and Stanley et al. [6,7] found that the trading volume will turn large when the market trend switches, but a trend switch is not sufficient for a full fledged crisis. Power law provides little information for timing a crisis since its data requirement implies a large time window in statistical analysis. Critical information on a crisis is not reliable from power law, since the tails could decay faster than power law. Although we may have better approximations of the sample distributions by means of other models, we still lack useful information on crisis warnings [8]. Methodologically speaking, a stable pdf in a long time window cannot offer a real-time monitor of the degree of market stability. For managing a financial crisis, we need an effective indicator in a short time window. Sornette [9] noticed that some large market crashes are outliers of a stable market (such as the crashes in Apr. 2000 and Oct. 1987). He identified a log-periodic pattern from a possible bubble buildup process. His problem is that he needs a theory to justify his model, since there is little evidence of harmonic waves from business cycle data. We found solid evidences of continuous-time color chaos with an erratic amplitude but a narrow frequency band from macro and stock indexes, which are nonlinear and aperiodic in nature [10, 11].

We developed a third approach, which reveals the degree of market instability and the timing of a coming crisis from a non-stationary time series analysis. Our numerical representation is high moments in statistics. Our theoretical framework is the master equation in statistical mechanics. Our simplified model for market price up–down movements is the population model of the birth–death process. The Master equation has been used in option pricing and herd behavior in the financial market [12,13]. Schrödinger's Principle of Large Numbers sheds light on market resilience from macro and finance data [14,15]. The birth–death process is the simplest population model, which ensures the Principle of Large numbers in stochastic dynamics [14]. Market instability and crises can be described by a regime switch in the nonlinear stochastic process. Our numerical experiments show that the high moment (3rd to 5th moment) returns from stock market indexes can be a useful indicator of economic complexity [16] and market instability. Non-linearity and complexity in economics and physics have many interesting features, such as power law, fractal, network, and criticality [5–7,17–21]. Understanding economic complexity will open new ways of research in financial economics.

This work shows that high moments reveal critical information on dynamical instability and crisis timing from a non-stationary financial time series. It is known that business cycles are non-stationary in nature. The 1st and 2nd moments reveal limited information of a changing market regime in volatility and returns [22]. Markowitz [23] realized that the second order moment is only feasible under equilibrium arbitrage and high moments may be the cause of market speculation, but he discarded higher than two moments because he could not figure out the economic interpretation for high moments at that time. We found out that the dramatic rising of higher than 2nd moments (especially the third to fifth moments) provides a clear indicator of a dynamic regime switch and a likely coming crisis. Our high moments approach has one advantage: conceptually, a statistical moment is clearly defined both in statistics and statistical mechanics, so that we can develop a unified approach in empirical analysis and theoretical modeling. In practice, there is an operational problem in calculating the moments. As we know, the mean, the cumulant, and probability distribution are stable only in controlled experiments in physics labs. Economic time series are not obtained from controlled experiments and they are non-stationary in nature. Therefore, numerical moments can only be calculated through a moving time window. Thus, the working definition of moments is an empirical issue in financial analysis. Its usefulness should be verified by its power in explaining observed patterns and historical events. We tested high moments measured by a short time-window of one period that serves as an indicator in studying critical phenomena in economics. We found that the dramatic rise (1000 times or more) of high (3rd to 5th) moments before and during a crisis, which may serve as the signal of a market breakdown [24]. Now we have a better understanding of why diversification strategy failed during the sub-prime crisis. The mean–variance approach ignores excess speculation driven by high moment deviations, which is significant before and during a crisis period. For diagnosing a crisis generated by herd behavior, we introduce a population dynamic of the birth–death process for describing up–down price movements by means of a stochastic differential equation. The Black–Scholes option pricing model based on the representative agent model of geometric Brownian motion can be extended to the more generalized population model of the birth–death process, we would discuss this issue elsewhere [25]. In this article, we will focus on diagnosing a financial crisis by means of high moments representation and identifying a crisis condition from the birth–death process, since social interaction in collective action is the source of herd behaviors and market fads in behavioral economics [26].

In the following sections, the limits of a stable distribution and the advantages of high-moments representation are discussed in Section 2. The up–down price dynamics described by the birth–death process and the high moment