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The role of information in a two-traders market

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HIGHLIGHTS

- We propose an operatorial technique in the analysis of a two trader system.
- We use lowering and raising operators to analyze the role of information for traders.
- The information is modeled as an external reservoir, interacting with the system.

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ABSTRACT

In a very simple stock market, made by only two *initially equivalent* traders, we discuss how the information can affect the performance of the traders. More in detail, we first consider how the portfolios of the traders evolve in time when the market is *closed*. After that, we discuss two models in which an interaction with the outer world is allowed. We show that, in this case, the two traders behave differently, depending on (i) the amount of information which they receive from outside; and (ii) the quality of this information.

1. Introduction and motivations

In a series of papers, [1–4], one of us (FB) has shown how the Heisenberg time evolution used for quantum mechanical systems can be adopted in the analysis of some simplified stock markets. After these original applications, the same tools were also used for rather different macroscopic systems. A recent monograph on these topics is [5]. In the cited papers and in Ref. [5] the role of information was, in a certain sense, only incorporated by properly choosing some of the constants defining the Hamiltonian of the system we were considering.

On the other hand, the other author (EH), following the original idea of Ref. [6], considered the role of information for stock markets, [7,8], mainly adopting the Bohm view to quantum mechanics, where the information is carried by a pilot wave function $\Psi(x, t)$, satisfying a Schrödinger equation of motion, and which, with simple computations, produces what is called *a mental force* which has to be added to the other *hard* forces acting on the system, producing a full Newton-like classical differential equation.

In this paper we try first to incorporate the effect of this mental force at a purely quantum mechanical level. After that, we consider a simplified stock market, which, to simplify the notation, we consider with just two traders τ_1 and τ_2, describing what happens *before* the trading begins, i.e. in the phase in which the information begins to circulate in the market, and is used by the traders to decide their next moves. The rationale for focusing on the way information can influence valuation

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of portfolios is a very important topic in finance and economics. We stress that it is the modeling of the information which is at the heart of the problem in such valuation exercises. We believe this paper shows that tools from quantum mechanics can aid in a very valuable way to this modeling challenge.

It may be worth stressing that our analysis continues on a nowadays rather rich literature on the role of quantum mechanics in economics, see Refs. [9–14] for instance, which shows that an increasing number of researchers believe that some of the aspects of a *real* stock market could be described by adopting tools and ideas coming from quantum mechanics. We should stress that, in our knowledge, the first paper where such a connection between quantum mechanics and finance appeared is [15], where the authors suggested that non commuting operators are really needed in the description of a realistic market to prevent exact knowledge of the price of a share and of its forward time derivative. See also Ref. [12]. These two quantities, in Refs. [12,15], were associated to operators having the same commutation rule as the position and the momentum operators in ordinary quantum mechanics, and therefore obey an uncertainty principle. Furthermore, there is scope to argue that for instance the central concept of non-arbitrage in finance has connections with hermiticity in quantum mechanics. Baaquie [12] has shown that the Hamiltonian of the Black–Scholes equation is not hermitian. This non-existence of hermiticity is narrowly related to the absence of arbitrage (the existence of a martingale). Clearly, hermiticity on itself is not making anything quantum mechanical as such, but it is still an important argument. There are other interesting arguments, such as the way hidden variable theory can connect with the (non-observable) state prices, in again, the non-arbitrage theorem. See Ref. [16]. Finally, we also want to mention that in the context of decision theory, notably in the resolving of some expected utility paradoxes, the use of quantum probability is very promising. Those paradoxes lie at the base of many economics/finance models. We document those achievements in Ref. [16]. In essence, the use of quantum mechanical techniques in social science revolves really around formalizing information. See Ref. [5].

The paper is organized as follows: in the next section we briefly discuss how the pilot wave function can be incorporated in our Heisenberg-like dynamics. Then, in Section 3 we introduce a first model of a closed market, where the information (or, in our setting, the *lack of information*, LoI in the following) will behave as the other operators, i.e., it will be described by ordinary two-modes bosonic operators. In Section 4 we replace these operators with two families of bosonic operators, describing sources and sinks of information which modify, in the way described below, directly the portfolios of the traders. In Section 5, finally, we consider a more complete model where the outer world contributes in the definition of the strategies of the traders in a more realistic way, i.e. by contributing to the information of the traders, rather than being the information by itself. Section 6 contains our conclusions.

2. Some preliminaries

In FB's approach to stock markets the essential ingredient of the model is the Hamiltonian operator H which is taken to describe the system. In Ref. [5] several useful rules have been proposed to fix the expression of H. We need now to incorporate in H the effect described by the pilot wave function, extending, for instance, what is discussed in Ref. [17]. See also Ref. [6]. Let us recall here the essential steps: the main ingredient is the (two-dimensional, in our case) pilot wave function, $\Psi(q_1,q_2)$, which evolves in time according to the Schrödinger equation of motion

$$\mathrm{i}\frac{\partial\Psi(q_1,q_2;t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\sum_{j=1}^{2}\frac{\partial^2}{\partial q_j^2}+V(q_1,q_2)\right]\Psi(q_1,q_2;t),$$

where $\hbar$ and m have a suitable economics based meaning,[1] [6,17], and $V(q_1,q_2)$ is the potential due to the hard economics based effects. Then, calling $R(q_1,q_2)=|\Psi(q_1,q_2)|$, a new potential is constructed by defining $U(q_1,q_2)=-\frac{1}{R(q_1,q_2)}\sum_{j=1}^{2}\frac{\partial^2 R(q_1,q_2)}{\partial q_j^2}$, and $U(q_1,q_2)$ produces the mental forces affecting the traders: $g_j(q_1,q_2)=-\frac{\partial U(q_1,q_2)}{\partial q_j}$, $j=1,2$. Please note the definition of this new potential is not foreign to physics but is squarely steeped into Bohmian mechanics (which is a particular interpretation of quantum mechanics). The key references are [18,19]. *We note that a lot of research has been published around* $U(q_1,q_2)$. *This term will for instance appear in the stochastic equivalent of the Hamilton–Jacobi equations* [20]. *See also Ref.* [21]. *The approach by Nelson is also linked to the so called Madelung equations* [22]. *The Bellman function in dynamic programming (an optimization technique well known in economics for instance), can also have a connection to Madelung equations. See Refs.* [23–26].

Finally, if we call $\pi_j(t)$ the value of the portfolio[2] of τ_j, its time evolution is driven by the following classical (Newtonian-like) differential equation:

$$\dot{\pi}_j(t)=-\frac{\partial V(q_1,q_2)}{\partial q_j}-\frac{\partial U(q_1,q_2)}{\partial q_j}=:f_j(q_1,q_2)+g_j(q_1,q_2),$$

$j=1,2$, with obvious notation. Hence, the time evolution of $\pi_j(t)$ is governed by hard factors (f_j) as well as by the *financial mental force* g_j, [6,17].

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[1] It is to be noted that to give an economics based interpretation of $\hbar$ is still a very difficult challenge.

[2] This approach is slightly different from Refs. [6,17], but it is more natural in the present context.