Physica A 404 (2014) 291–295

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Characterizing vertex-degree sequences in scale-free networks[☆]

Wenjun Xiao [a,*], Yanxia Liu [a], Guanrong Chen [b]

[a] School of Software Engineering, South China University of Technology, Guangzhou 510006, Guangdong, China
[b] Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong Special Administrative Region

ARTICLE INFO

Article history:
Received 23 October 2013
Available online 4 March 2014

Keywords:
Complex network
Scale-free network
Power-law distribution
Vertex-degree sequence

ABSTRACT

Many large-scale complex networks exhibit a scale-free vertex-degree distribution in a power-law form. To better understand the mechanism of power-law formation in real-world networks, we explore and analyze the underlying mechanism based on the vertex-degree sequences of such networks. We show that for a scale-free network of size N, if its vertex-degree sequence is $k_1 < k_2 < \cdots < k_l$, and if its power exponent satisfies $\gamma > 1$, then the length l of the vertex-degree sequence is of order $\log N$. We verify this conclusion by a co-authorship network and some other real networks in various areas.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

Complex networks are ubiquitous in nature and human society; examples include the Internet, the World Wide Web, protein networks, scientists' collaboration networks and transportation networks.

Recently, new interests in the new theory and models of complex networks started from the findings of the small-world network model [1] and scale-free network model [2]. Scale-free networks, in particular, have a heterogeneous vertex connectivity, in which a small fraction of vertices are highly connected. The scale-free network model of Barabási and Albert [2] demonstrated the essential power-law distribution of vertex degrees, in the form of $P(k) \propto k^{-\gamma}$ where k is the degree variable and γ is a constant, which is a direct consequence of two generic mechanisms that govern the network formation: (i) network expansion over time through addition of new vertices; (ii) preferential attachment of new vertex to those existing ones that are already highly connected in the network.

In this paper, we focus on scale-free networks. It is now well known that most real scale-free networks have $\gamma \geq 2$ by default [3–8], but there are also many real scale-free networks that have $\gamma < 2$ [9–12]. It is meaningful to study the similarities and differences between such networks with $\gamma \geq 2$ and $\gamma < 2$, respectively. In our previous works [7,8], we have presented some necessary conditions for scale-free property formation in complex networks, based on the assumption of $\gamma \geq 2$. Here, we further extend our results to the case of $\gamma < 2$ and, in particular, scale-free networks with exponents $\gamma > 1$.

Specifically, in this paper we will show that for a scale-free network of size N, if its vertex-degree sequence is $k_1 < k_2 < \cdots < k_l$, and if its power exponent satisfies $\gamma > 1$, then the length l of the vertex-degree sequence is of order $\log N$. We will verify this conclusion by a co-authorship network and some other real networks in various areas.

☆ Project supported by the National Nature Science Foundation of China (Nos. 61170313, 61103037, and 61370003) and Ph.D. Programs Foundation of Ministry of Education of China (No. 20110172110028).

* Corresponding author.
E-mail address: wjxiao@scut.edu.cn (W. Xiao).

http://dx.doi.org/10.1016/j.physa.2014.02.036
0378-4371/© 2014 Elsevier B.V. All rights reserved.

2. Scale-free networks and previous results

A complex network can be represented by an undirected or a directed graph, $G(V, E)$, where V is the set of vertices and E is the set of edges. A graph has a number of local and aggregate parameters that characterize its structure (e.g., regularity, modularity, dimensionality), connectivity (e.g., density, diffusion) and robustness (e.g., resilience to random or malicious faults).

The following list summarizes the main parameters to be used in this paper:

- M — Number of edges; $M = |E|$
- N — Number of vertices; $N = |V|$
- $\mathrm{d}(v)$ — Degree of vertex $v \in V$
- $\overline{d}$ — Average vertex degree of the network; $\overline{d} = 2M/N = \sum_{v\in V} \mathrm{d}(v)/N$
- n_k — Number of degree-k vertices: $n_k = \{v|\mathrm{d}(v) = k\}$
- $P(k)$ — Degree distribution, or fraction of vertices of degree k: $P(k) = n_k/N$
- l — Length of vertex-degree sequence $k_1, k_2, \ldots, k_l$, $1 \le k_1 < k_2 \ldots < k_l$.

For scale-free networks, we have

$$P(k) = ck^{-\gamma}, \quad \gamma > 1.$$

Here, the requirement of $\gamma > 1$ ensures that $P(k)$ can be normalized. The constant c is used for normalization, $c = \left(\sum_{k\in K} k^{-\gamma}\right)^{-1}$, in which K is the set of all vertex degrees in the network.

In our previous works [7,8], we proved that for a scale-free network of size N, having a power-law distribution with exponent $\gamma \ge 2$, the number of degree-1 vertices, if not zero, tends to be of order N and we also proved that the average degree is of order lower than $\log N$. Our method provides an analytical tool that helps to check if a given network is scale-free because it relies on static conditions that are easily verified. Furthermore, we showed that the number of degree-1 vertices is divisible by the least common multiples $k_1^{\gamma}, k_2^{\gamma}, \ldots, k_l^{\gamma}$, where $k_1 < k_2 < \cdots < k_l$ is the vertex-degree sequence of the network. This leads a remodeling method to equip a scale-free network with small-world features.

In this paper, based on our previous results [7,8], we will show that for the above scale-free networks with $\gamma > 1$, the length l of the vertex-degree sequence is of order $\log N$. We will also verify this conclusion by a co-authorship network and some other real networks in various areas.

3. A new characteristic of scale-free networks and its derivation

3.1. Lengths of vertex-degree sequences in scale-free networks

In this section, we present a new characteristic of scale-free networks and its mathematical derivation. Supposing that the vertex-degree sequence of network is $1 \le k_1 < k_2 < \cdots < k_l$. According to the definition of scale-free networks, we have

$$P(k_i) = \frac{n_{k_i}}{N} = ck_i^{-\gamma}. \tag{1}$$

Here, n_{k_i} is the number of vertices with degree k_i, satisfying

$$N = \sum_{i=1}^{l} n_{k_i} \tag{2}$$

and c is a normalizing constant: when $i = 1$, we have

$$P(k_1) = \frac{n_{k_1}}{N} = ck_1^{-\gamma}. \tag{3}$$

Thus,

$$c = \frac{n_{k_1} k_1^{\gamma}}{N}. \tag{4}$$

By substituting (3) into (1), we obtain

$$n_{k_i} = n_{k_1}\left(\frac{k_1}{k_i}\right)^{\gamma}. \tag{5}$$

It follows from (2) and (5) that

$$N = \sum_{i=1}^{l} n_{k_i} = n_{k_1} k_1^{\gamma} \sum_{i=1}^{l} \frac{1}{k_i^{\gamma}}. \tag{6}$$

Download English Version:

https://daneshyari.com/en/article/7381393

Download Persian Version:

https://daneshyari.com/article/7381393

Daneshyari.com