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Multifractals of central place systems: Models, dimension spectrums, and empirical analysis

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HIGHLIGHTS

- Two multifractal central place models are presented for spatial analysis of cities.
- Central place multifractals can model real spatial features of systems of cities.
- Geographical spatial processes can be divided into concentration and deconcentration.
- Real systems of cities and towns take on multifractal structure.
- The values of multifractal dimension spectrums of urban systems range from 0 to 2.

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ABSTRACT

Central place systems have been demonstrated to possess self-similarity in both the theoretical and empirical perspectives. A central place model can be treated as a monofractal with a single scaling process. However, a real system of human settlements is a complex network with multi-scaling processes. The simple fractal central place models are not enough to interpret the spatial patterns and evolutive processes of urban systems. It is necessary to construct multi-scaling fractal models of urban places. Based on the postulates of intermittent space filling and unequal probability of urban growth, two typical multifractal models of central places are proposed in this paper. One model is put forward to reflect the process of spatial concentration (convergence), and the generalized correlation dimension varies from 0.7306 to 1.3181; the other model is presented to describe the process of spatial deconcentration (divergence), the generalized correlation dimension ranges from 1.6523 to 1.7118. An empirical analysis was made by the cities and towns of Central Plains, China, and an analogy is drawn between the real system of urban places and the theoretical models. A finding is that urban systems take on multifractal form, and can be modeled with multi-scaling fractals. This is a preliminary attempt to develop the theory of fractal central places, and the results are helpful for understanding the similarities and differences between the dynamical process of spatial concentration and that of spatial deconcentration.

1. Introduction

Fractal geometry represents a new paradigm for geographical studies, and in recent years, the impact of fractal ideas upon geography becomes more significant than ever. In fact, many of traditional theories in both physical and human geography are being reinterpreted using concepts and principles from fractals. The most important theory in human geography is the models of central places, which can be associated with fractal patterns. Central place theory was created by Ref. [1],

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and it seeks to explain the size, number, and location of human settlements in a regional system [2]. A central place is a settlement or a nodal point of transport network that, by its functions, serves an area round about it for goods and services. To ensure that goods and services are freely available, higher central places emerge at the center of hexagon containing six lower-order central places [3]. In a sense, a central place system is a network of cities and towns linked together in the form of integrated spatial hierarchy of centers of different functions and sizes [4]. The basic features of central place systems are triangular lattice pattern, regular hexagonal boundaries, and nested hierarchical structure. The spatial texture of a central place network was theoretically proved to be fractal [5–7], while the spatial structure of a central place system was empirically demonstrated to be of self-similarity [8]. The number, size, and distance between different locations of human settlements can be formulated as three exponential laws, from which it follows three power laws indicative of allometric scaling and fractals [9]. Central place fractals belong to fractal cities [10]. The introduction of fractal ideas into central place theory indicates a significant progress of human geography.

A problem arises that simple fractal central place models are not sufficient for explaining and predicting the patterns and processes of complex systems of human settlements in the real world. The original models of central places presented by Ref. [1] are based on the assumptions of unbounded isotropic, homogeneous, limitless surfaces with evenly distributed population and resources, which lead to the equal probability of urban growth in geographical space. In spite of different population sizes, the probabilities of urban emergence in different places are equal to one another. This kind of model can be generalized to *monofractals* of central place systems, which are based on one scaling process. However, the actual settlement systems always develop on heterogeneous surfaces with unevenly distributed population and resources, which result in the unequal spatial probability of growth of cities and towns. Just because of this, Christaller's central place models were subsequently improved by Refs. [11,12] and others [13,14], who no longer took an evenly distributed set of urban places for granted. The unequal probability of growth of human settlements suggests that central places systems should be described with *multifractals*, which are based on multi-scaling processes. Frankhauser [15] once proposed a preliminary multifractal model of transport network in terms of central place patterns, but the further research remains to be made with fractal geometry. It is necessary to apply the ideas from multifractals to central place systems to advance the theory and method of spatial analysis [16].

There are two approaches to studying central place fractals. One is the theoretical approach, and the other, the empirical approach. The former starts from model building, while the latter proceeds from data analysis. In fact, any mathematical tool has two functions for scientific researches: one is to process experimental or observational data, and the other is to construct postulates, make models, and develop theory. If the fractal concepts are employed to build mathematical models for cities as systems or systems of cities, it is a theoretical approach; if the fractal geometry is used as a means of data processing for urban systems, it is an empirical approach. In geography, generally speaking, the fractal-based theoretical approach comprises the following steps: defining postulated conditions, proposing/deriving fractal models, and, if possible, testifying/verifying the models, revealing laws, rules or principles, and the aim is to propound a theory. The empirical approach consists of the below steps: defining a study area, obtaining observed data, computing fractal dimensions, making an analysis with the fractal parameters, and the aim is to bring to light the temporal processes and spatial patterns of geographical systems. In many cases, the two approaches can be utilized in the same study to supplement one another.

So far, multifractals theory has been applied to human geographical studies through both theoretical and empirical approaches, and the results are interesting and revealing [16–21]. In this paper, the theoretical approach is employed to research central place fractals. Two multi-scaling fractal models of central places will be presented. One is to reflect spatial concentration of urban places, and the other is to mirror spatial deconcentration of cities and towns. Concentration and deconcentration represent two typical spatial patterns, reflecting two extreme evolutive processes of geographical systems. The rest parts are organized as follows. In Section 2, theoretical postulates, geometrical models, and dimension spectrums are presented. Then, in Section 3, an analogy is drawn between the fractal models and the real patterns of human settlements in Central Plains, China. A number of questions are discussed in Section 4, and finally, the paper is concluded with a brief summary. The main significance of this study is that new models of multifractals are constructed for urban studies, and the models are useful for our understanding the geographical processes and patterns of human settlements.

2. Multifractal models

2.1. Basic postulates and main parameters

The classical models of central places are based on the growth processes of equal probability and intermittency-free space filling. In this case, the human settlements in the same class are evenly distributed over an area. Despite the self-similar texture consisting of fractal lines [5], the theoretical dimension of spatial structure of central place systems is $d = 2$, which is a Euclidean dimension [9]. However, the actual patterns of cities and towns are not spatially homogeneous, and the empirical dimension is a fraction rather an integer [8]. In other words, a central place landscape is a fractal-like pattern in the real world. In order to present the multifractal models of central places, two postulates are given as follows. First, the space filling of central place development is a process of intermittency. The concept of intermittency indicates alternate containing and emptying of human activities in a geographical region due to unevenly distributed resources. Second, the growth of human settlements is of unequal probability in geographical space. That is, different places have different probabilities of urban emergence because of different geographical conditions.