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Individual behavior and social wealth in the spatial public goods game

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HIGHLIGHTS

- We introduce star-like graphs to demonstrate why selective investment mechanism favors cooperation.
- Selective investment mechanism reinforces the positive/negative feedback mechanism for hubs.
- High-degree nodes behave inert, whereas low-degree nodes are active and always influenced by others.
- Social wealth of Public Goods Game on scale free networks follows a power law distribution.
- The enhancement factor r in the Public Goods Game model can regulate social inequality.

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ABSTRACT

Group interactions on structured populations can be represented by the public goods game on networks. During the evolutionary games, selective investment mechanism fosters social cooperative behavior. First we focus on star-like graphs to provide some light on why selective investment mechanism can promote collective cooperation. Then we implement public goods game with this mechanism on scale free networks to investigate behavior properties of individuals within different social environments. We indicate that high-degree nodes are predominantly inert owning largely to their satisfaction with their status, while low-degree nodes are very active due to their strive towards higher prosperity. Besides, we introduce the Gini coefficient to describe social inequality and find that large multiply factor r favors social fairness. Our work is applicable for community supervision and social wealth regulation.

1. Introduction

Evolutionary game theory [1] has become a valuable approach to study collective cooperative behavior during the last years. Some studies on evolutionary game theory consider the prisoner's dilemma game to investigate cooperation phenomenon [2,3]. Prisoner's dilemma game describes a situation in which cooperation is always hampered by the temptation of cooperators to defect and by selfish defectors. It leads to a social dilemma since unanimous cooperation brings about higher benefit than mutual defection. When populations are assumed to be infinite and well-mixed, replicator equation and best response, together with related learning dynamics [4] can be formulated for the evolution of collective cooperation. However, this assumption of population homogeneity is unrealistic. Considering the fact that real populations are never well-mixed and infinite, many researchers turn to investigate a cooperation evolution set on certain graphs which

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represent geographical isolation or social networks [5–7]. In a graph, node represents individual and link represents relationship. In such situation, the evolution of collective cooperation is specified that each individual can interact only with its direct neighbors, and that strategy updating can take place only along the edges of this graph. An important representative of collective interaction on graphs is the m-player generalization of prisoner's dilemma game, also called the public goods game [8]: cooperators (C) contribute an amount c to the public goods, defectors (D) contribute nothing. All the contributions are then multiplied by an enhancement factor r to take into account synergetic effect of cooperation, and the results are equally distributed among all m numbers of this game. Each individual i of degree k_i plays $k_i + 1$ games centered on itself and its k_i neighbors separately, and it contributes to each game without bias. This homogeneous contribution strategy can be called homogeneous investment mechanism.

For the evolution of public goods game played on graphs, scientists have proposed many explanations to figure out the emergence and maintenance of collective cooperation, such as rewards and punishments [9–14], feedback reciprocity mechanism [15,16], risk of collective failure [17–19], strategy diversity [20] and social heterogeneity [21–25]. For example, M. Perc et al. demonstrate that the effectiveness of punishment can be significantly elevated through invigorating spatial reciprocity and preventing the emergence of strategy cyclic dominance [14]. Z. Rong et al. indicate that the feedback reciprocity mechanism from triangle loops supports mutual cooperation via resisting invasion of selfish behavior [16]. Besides, the risk of collective failure can also stimulate people to contribute [17,18] or move away from unfavorable location [19], since all members will lose their endowments with a probability if contributions to the common goods are too small. A. Szolnoki et al. employed conditional strategy (willingness to cooperate depends on the behavior of others) to show that conditional cooperators can force defectors into isolated convex bubbles from which they are unable to exploit the common goods [20]. Santos et al. introduce social heterogeneity by implementing public goods game on heterogeneous networks [21], and they demonstrate that the diversity of game number and group size plays a crucial rule in promoting cooperation. Remarkably, M. Perc et al. have published a review of recent advances in the evolutionary dynamics of spatial games governed by group interactions [26].

In this paper, we focus on social heterogeneity by adopting selective investment mechanism generalized from Ref. [23]: individuals invest more to those groups of high-quality, here defined as cooperation level. It can describe the fact that when making investment decision, rational individuals always prefer certain high-quality groups whereas exclude others. By means of numerical simulations, it has been shown that investment heterogeneity from selective investment mechanism can promote cooperation. However, in addition to numerical simulations, we examine the intrinsic motivation by analyzing the public goods game played on star-like graphs [21,27,28]. Besides, we investigate the behavior properties of individuals within different neighborhoods on top of Barabási–Albert (BA) scale free networks [29]. In addition, we adopt the Gini coefficient [30] to analyze the social wealth and determine how to improve social fairness.

2. Model description

For better understanding, here we give a more thorough description of the model. We consider the evolution of public goods game with selective investment mechanism on BA scale free networks, whose number of nodes is set as $N = 1000$, and the average connectivity as $\langle k\rangle \simeq 4$. Initially each individual is designed either as a cooperator or a defector with equal probability. At each time step t, each individual i with degree of k_i plays public goods games within the $k_i + 1$ groups to which it belongs (using the same strategy in each game). Simply, player i has k_i neighbors, so it participates in $k_i + 1$ games: $G_1, G_2, \ldots, G_{k_i+1}$. Individual can acquire the information of cooperation level in the last round of each game $q_j(t-1) = n_j(t-1)/(k_j+1)$, where $n_j(t-1)$ is the number of cooperators in group G_j in the last round game. Then cooperator i contributes $U_{ij}(t)$ to game G_j, which is

$$U_{ij}(t) = c\frac{q_j(t-1)}{\sum\limits_{s=1}^{k_i+1} q_s(t-1)}. \tag{1}$$

Group G_j collects investment from all cooperators. Then the results are enhanced by a factor r, next equally distributed to all members. Therefore the profit that cooperator i obtains from G_j is

$$P_{ij}(t) = r\frac{\sum\limits_{k\in\Omega_j} U_{kj}(t)}{N_j} - U_{ij}(t), \tag{2}$$

where Ω_j is the set of cooperative neighbors of j, and N_j denotes the size of group G_j. Accordingly, the payoff difference between C and D within group G_j is $U_{ij}(t)$. The overall payoff of cooperator i is

$$P_i(t) = \sum_{j=1}^{k_i+1} P_{ij}(t). \tag{3}$$

After each process of investment and distribution, all the individuals update their strategies synchronously according to Fermi function [31]: each individual i chooses a neighbor j randomly, and the chosen neighbor j also acquires its payoff P_j in