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The effect of zealots on the rate of consensus achievement in complex networks

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HIGHLIGHTS

- We investigate the role of zealots on the result of voting process on both SF and WS networks.
- Increasing the zealot's number Z, increases the rate of consensus achievement on both of the networks.
- Increasing Z, exponentially reduces the time needed for the system to reach an ordered state.
- Increasing the re-wiring probability of a WS network, the efficiency of zealots is increased.

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ABSTRACT

In this study, we investigate the role of zealots on the result of voting process on both scale-free and Watts–Strogatz networks. We observe that inflexible individuals are very effective in consensus achievement and also in the rate of ordering process in complex networks. Zealots make the magnetization of the system to vary exponentially with time. We obtain that on SF networks, increasing the zealots' population, Z, exponentially increases the rate of consensus achievement. The time needed for the system to reach a desired magnetization, shows a power-law dependence on Z. As well, we obtain that the decay time of the order parameter shows a power-law dependence on Z. We also investigate the role of zealots' degree on the rate of ordering process and finally, we analyze the effect of network's randomness on the efficiency of zealots. Moving from a regular to a random network, the re-wiring probability P_{rw} increases. We show that with increasing P_{rw}, the efficiency of zealots for reducing the consensus achievement time increases. The rate of consensus is compared with the rate of ordering for different re-wiring probabilities of WS networks.

1. Introduction

One of the simplest models for dealing with cooperative behavior of an agent based system, is the *voter model*. This model is capable to be used as a paradigm for the dynamics of opinions in socially interacting populations. In general, the voting process is between two competing candidates or two opposite opinions with an equal initial inclination for each of candidates or opinions. In the original voter model, the society is modeled as a hyper-cubic lattice with N socially interacting nodes (vertices of the graph) and each node represents one voter with two possible opinion states: $+1$ and -1 [1–3]. The nodes on each lattice site, interact with their nearest neighbors. Two common methods for updating the nodes' opinions, are the *node-update* and *link-update*. One dynamical step under node-update dynamics is to choose one random node and assigning to it the opinion of one of its nearest neighbors, which is chosen at random too [3,4]. One physical time step corresponds to updating N nodes (N is the system size) so that each node is on average updated once. On the other hand, one dynamical step

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under link-update is randomly choosing a pair of nearest-neighbor spins, i.e. a link, and randomly assigning to both nearest-neighbor spins the same opinion when they had different opinions, and leaving them unchanged otherwise [3]. These two different updating methods lead to different conservation-law classes. In $d = 1$, node-update dynamics is equivalent to the zero-temperature *Glauber kinetic Ising model* [3,5,6]. Moreover, it has been shown that the node-update rule does not conserve the mean magnetization of the systems with heterogeneous degree distributions but, link-update rule does i.e. in the case of heterogeneous networks with an equal initial populations of two opposite signed spins, the mean magnetization always has a conserved value of $\langle m\rangle = 0$ in link-update but it may raise or fall from $\langle m\rangle = 0$ with node-updating. For homogeneous networks and also the small-world networks, because of the similarity of nodes in degree distribution, this difference is neglectable [3]. In the case of regular lattices, it has been shown that after a so-called *survival-time* τ, a finite size system reaches an ordered state in which the *interface density* decreases to zero, so that the survival time scales with the system size as $\tau \propto N^{\gamma}$ [3]. Numerical simulations obtain $\gamma = 1$ for regular hyper-cubic lattices [3] and, an analytical solution of voter-model on an annealed small-world network shows that $\gamma \simeq 1$ [3,7,8]. In the case of *Barabási–Albert* scale-free networks, a network with heterogeneous degree distribution, there is a difference between the results of two updating rules: node-update, which does not conserve the mean magnetization, gives $\gamma \simeq 0.88$ but, if the mean magnetization is conserved γ is obtained to be equal to 1 as for the regular lattices. Link update and also the *weighted node update* conserve the mean magnetization in both homogeneous and heterogeneous networks [3,8].

The role of *zealots*, individuals with inflexible opinions which are reluctant to change their opinions under any social interactions, has attracted much attention [9–13]. In one of the initial works in this area [10], the author has shown that existence of only one zealot individual in a *d* dimensional hyper-cubic lattice will lead the system to a stable state in which all the spins are parallel to the zealot's spin. The long time magnetization in 1, 2, and 3 dimensional system, as a function of time *t*, is proportional to $\sqrt{t},\ t/\ln(t)$, and t respectively [10]. In bare voter-model, the initial concentrates of each spin is the same and so, the dynamics is moved towards either one of two pure attractors. But the existence of inflexibles for only one of the two opinions, is found to change the initial needed balance into a lower value than 50% in favor of that side. On the other hand, analytical solution of three-party constrained voter model (with two radical species *A* and *B*, centrists as susceptibles and, a fixed fraction of zealous centrists) on complete graphs has shown that in an infinitely large population, there is a continuous transition between two different phases: a coexistence phase, which is stable when the fraction of centrist zealots is below the critical threshold, and a phase in which the fraction of centrist zealots is above the critical threshold and, centrism prevails [11].

The *density of interfaces*, *r*, characterizes the order parameter in a system with voter-model dynamics. In thermodynamic limit, when $N \to \infty$, the density of interfaces *r* depends on time *t* as $1/\sqrt{t},\ 1/\ln(t)$, and $(a - bt^{-d/2})$ in 1, 2, and $d > 2$ dimensional systems respectively [14,15]. For infinitely large systems, *r* finally reaches a saturated value in which the spins of the system continuously change their signs, although all of them are not parallel. So a completely ordered state is not possible for infinitely large systems [14]. In contrast, for a finite system, fluctuations will lead the system towards a full ordered state with global magnetization equal to $+1$ or -1 [3,14]. In a finite system, the only stable state is the case in which all the spins are paralleled and hence, *r* tends to zero (see Section 2). The solution of the voter model in the mean-field limit and on a one dimensional periodic ring (regular, and complete graph) has shown that having a few number of zealots is quite effective in inanimating the steady state in which consensus is never achieved [9]. But if an equal number of zealots of each type are present in the network, the system acts similar to the case in which there are no zealots, so that the final magnetization of the system will tend to one of its pure attractors -1 and $+1$ [9]. In the case of equal initial population for each of opinions, the existence of zealot voters for one of the opinions guarantees the winning of that opinion. However, for imbalanced initial state (in which the population of the opinion opposite to zealots is greater than the other) there will exist an incompressible minority around the zealots on the network. In this case, if the fraction of zealots is less than a critical value, the opinion opposite to the zealots will be the majority, but there will exist a stable minority of the other opinion (zealot's opinion). Beyond the threshold fraction of zealots, the opinion opposite to zealots always loses the election with no minority committed to them [16]. The role of network's average degree has been shown to be effective on the average interface density of the network [8,13] so that, one of the important ingredients in ordering process is the effective dimensionality of the network. Analyzes of the ordering dynamics of the voter-model in different classes of complex networks, show that the voter dynamics orders the system depending on the effective dimensionality of the networks and also, when there is no ordering in the system, the average survival time of metastable states in finite networks decreases with network disorder and degree heterogeneity. The existence of hubs in the network modifies the linear system size scaling law of the survival time [8]. All of these works and other works about the role of zealous individuals on the result of voting process, motivated us to analyze the role of zealots on the rate of consensus achievement on complex social networks.

In this work, we will discuss the role of zealots, individuals with fixed opinion, on both the mean magnetization and mean interface density in the case of scale-free and as well as Watts–Strogatz complex networks. We investigate the role of zealots on both the rate of consensus achievement and on the rate of ordering process in scale-free (SF) and Watts–Strogatz (WS) networks. In the next section, we will explain an extended voter model which is a modified version of original voter model to include the zealotry effects. Then, results of this model on SF and WS networks will be discussed in Section 3.

2. Model

In this work, we use an extended version of voter model in which the *zealots*, individuals that never change their opinions, are included in the population as well as *susceptible* individuals. The population with *N* voters is represented as a graph