Physica A 400 (2014) 159–167

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

On the computational complexity of the empirical mode decomposition algorithm

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HIGHLIGHTS

- The order of the computational complexity of the EMD is equivalent to FFT.
- Optimized program is proposed to speed up the computation of EMD up to 1000 times.
- Fast HHT with optimized EMD/EEMD algorithm can operate in real-time.

ARTICLE INFO

Article history:
Received 30 October 2013
Received in revised form 30 December 2013
Available online 21 January 2014

Keywords:
EMD
EEMD
Time
Space
Complexity

ABSTRACT

It has been claimed that the empirical mode decomposition (EMD) and its improved version the ensemble EMD (EEMD) are computation intensive. In this study we will prove that the time complexity of the EMD/EEMD, which has never been analyzed before, is actually equivalent to that of the Fourier Transform. Numerical examples are presented to verify that EMD/EEMD is, in fact, a computationally efficient method.

1. Introduction

EMD [1] is a nonlinear and nonstationary time domain decomposition method. It is an adaptive, data-driven algorithm that decomposes a time series into multiple empirical modes, known as intrinsic mode functions (IMFs). Each IMF represents a narrow band frequency–amplitude modulation that is often related to a specific physical process. For signals with intermittent oscillations, one intrinsic mode can comprise oscillations with a variety of wavelengths at different temporal locations. Collectively, the simultaneous exhibition of these disparate oscillations is known as the mode mixing phenomenon, which can complicate analyses and obscure physical meanings. To overcome this problem, the EEMD algorithm [2] and the noise-assisted MEMD [3] have been proposed. In particular, the EEMD has drawn a lot of attention

Abbreviations: EMD, empirical mode decomposition; EEMD, ensemble empirical mode decomposition; IMFs, intrinsic mode functions; MEMD, multivariate empirical mode decomposition; ADD, addition; MUL, multiplication; DIV, division; COMP, comparison; BFV, blood flow velocity signal.

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http://dx.doi.org/10.1016/j.physa.2014.01.020

Table 1
EMD.

```
[c_m] = Main (NS, n_m, y_0)

    r(t) = y_0(t)
    for (m = 1 : n_m) {   % extract each IMF
        x(t) = r(t)      % x(t) is proto-IMF
        for (s = 1 : NS) {   % sifting iteration:
            [t_max, x_max, t_min, x_min] = identify_extrema(x);
            U(t) = spline(t_max, x_max);      % Upper envelope
            L(t) = spline(t_min, x_min);      % Lower envelope
            x(t) = x(t) − (U(t) + L(t))/2;      % detail extraction      (1)
        }
        c_m(t) = x(t)                                                    (2)
        r(t) = r(t) − x(t)                                               (3)
    }
```

during the past few years. During the last decade, the EMD/EEMD was shown to be more effective than the traditional Fourier method in many problems from various fields such as physics [4], biomedicine [5–8], mechanical health diagnosis [9], image analysis [10] and others.

It seems a common belief that a major drawback of the EMD/EEMD is that they require a long computation time. For example, in Ref. [11], the authors report that it takes more than an hour to perform EMD on a signal with 30,000 data points using a modern personal computer. The enormous amount of computation time hampers the applications of EMD/EEMD in analyzing long data and pseudo real-time hardware applications. For this reason, parallel computing such as Cuda [12], Open MP [13] and hardware implementations like VLSI [14] and FPGA [15] have been recently adopted to improve the execution time.

In this study, we proved that the EMD/EEMD is actually not computation intensive as has been claimed in the past. We first analyze the time and storage complexity for EMD/EEMD and their variants in different applications under an appropriate implementation, which is based on single core, sequential implementation without any approximations. Finally, numerical examples are presented to verify the performance of EMD/EEMD.

The remainder of this article is divided into the following sections: in Section 2, we briefly review the EMD method. In Section 3 we present an implementation of the EMD/EEMD method and then analyze their time and storage complexities. Section 4 illustrates the efficacy of the proposed method with experimental results and we conclude our study in Section 5.

2. The EMD algorithm

Given an input signal $y_0(t),\ y_0 \in R,\ t \in Z$ and $t = [1 : n]$, the EMD decomposes a signal $y_0(t)$ into a series of intrinsic mode functions (IMFs) which are extracted via an iterative sifting process. First, the local maxima and minima of the signal are connected, respectively, by cubic splines to form the upper/lower envelopes. The average of the two envelopes is then subtracted from the original signal. This sifting process is then repeated several times (define the number of siftings as *NS*, usually 10) to obtain the first IMF that oscillates at relatively higher frequencies than does the residual signal that is obtained by subtracting the IMF from the original signal. Then, the residual is deemed as the input for a new round of iterations. In turn, subsequent IMFs with lower oscillation frequencies are derived using the same process and the newly obtained residue. The result of the EMD is a decomposition of the signal $y_0(t)$ into the sum of the IMFs and a residue $r(t)$. That is,

$$y_0(t) = \sum_{m=1}^{n_m} c_m(t) + r(t)$$

where n_m is the number of IMFs. The EMD algorithm is listed in Table 1.

3. Time and space complexity

3.1. EMD

In this section, we propose an implementation of the EMD and analyze its time and space complexity. The arithmetic operations involved include addition (*ADD*), multiplication (*MUL*), division (*DIV*) and comparison (*COMP*). Fixed-point arithmetic operations are negligible compared with the floating-point operations. To facilitate the analysis, the EMD procedure is divided into the main, extrema identification and cubic spline procedures. Capital T and M denote the time and space (storage) complexity, respectively. The EMD algorithm is listed in Table 1 and is explained in detail below.

3.1.1. Main procedure

Each of the input, residue $r(t)$, proto-IMF $x(t)$ and the upper/lower envelope $U(t)/L(t)$ is of length n and requires n floating point storage. The output signal consists of n_m IMFs each with length of n, hence it requires $n_m \cdot n$ floating-