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Firm competition in a probabilistic framework of consumer choice

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HIGHLIGHTS

- We use a probabilistic consumer choice framework to model firm competition in a market.
- Firms are allowed to compete by both quality and price of their products.
- We find Nash market equilibria in various situations.
- Contrary to classical Bertrand competition, marginal firm profits do not always vanish.
- Small firms are able to improve their profit by engaging in competition.

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ABSTRACT

We develop a probabilistic consumer choice framework based on information asymmetry between consumers and firms. This framework makes it possible to study market competition of several firms by both quality and price of their products. We find Nash market equilibria and other optimal strategies in various situations ranging from competition of two identical firms to firms of different sizes and firms which improve their efficiency.

1. Introduction

Firm competition, one of the most basic market processes, has been famously discussed by Adam Smith [1]. Two pioneering models by Cournot and Bertrand [2, Chapter 27] then described firm competition by quantity and price, respectively, and provided the first explanations of market behavior in their respective cases. In the Bertrand model, consumers give absolute preference to the lowest price which consequently drives firm profits to zero. By contrast, the Cournot model assumes that the offered products are homogeneous (indistinguishable), derives the price from the aggregate quantity produced by all firms, and allows non-zero profits to be made. While it may seem obvious that Bertrand competition is more beneficial for the consumers than Cournot competition, this is not always the case [3]. Firm competition models were later improved by modeling the consumer choice through a utility function which is maximized by each individual consumer and whose maximum then reflects the market's behavior. An example of this approach is provided by the classical Dixit–Stiglitz model of monopolistic competition [4] and an extensive line of work that it has inspired [5].

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It soon became clear that a certain degree of price dispersion is present in real markets [6,7] and thus models building on the assumption of a unique price are insufficient. A market where both informed and uninformed customers are present was shown to lead to "spatial" price dispersion where some stores sell at a competitive price and others sell at a higher price [8]. The phenomenon of "temporal" price dispersion where each store varies its price over time (and thus prevents the customers from learning and distinguishing "good" and "bad" shops) has been modeled in Ref. [9]. See Ref. [10] for an exhaustive review of work on price dispersion. However, even these models based on the economics of information [6] and the search of consumers for information in a market [11] are not entirely satisfactory because they assume that upon inspection, a consumer is able to exactly determine the utility of a given product.

We build on a market model where each product is endowed with intrinsic quality and each consumer with quality assessment ability (in general, both quality and ability are continuously distributed over a certain range) [12,13]. The demand is generated by consumers, not imposed by firms. This model was shown to produce product differentiation where high-quality products target experienced consumers and low-quality products target the unexperienced (or negligent) ones [14]. While [14] deals with the case of heterogeneous consumers served by a monopolist firm, we now focus on homogeneous consumers served by multiple firms. By assuming that each consumer has a maximal price which they are willing to pay, we generalize this framework to include also product price in the consumer decision process. This allows us to model firm competition by product quality and price. With respect to other works where, typically, two consumer groups and two different product levels distinguished by quality or price are considered [15], the current framework makes it possible to explicitly study the impact of consumer ability on the market equilibrium. It contributes to an extensive line of complex systems research which has helped to understand basic features of various systems in economics [16,17], sociology [18], and network science [19].

The rest of this paper is organized as follows. In Section 2, we introduce the framework and find the optimal strategy of a monopolist firm. In Section 3, we study the basic case of two homogeneous firms competing in the market, characterize the market (Nash) equilibrium upon various strategies adopted by these firms, and show that firms can outperform the Nash equilibrium. In Section 4, we study three simple generalizations of the basic case: competition of several firms, competition of firms of different size, and the effect of unequal firm efficiency on the market. In Section 5, we summarize the work and discuss the most important open questions and further research directions.

2. Basic framework

We present here a probabilistic framework for consumer choice and firm profit which deviates from the framework studied in Ref. [14] by considering price sensitivity of consumers. The two basic characteristics of any product offered by a firm are its intrinsic quality Q and its price p. The probabilistic behavior of the consumers is due to information asymmetry between them and firms which results in consumers being unable to assess quality of products with certainty. The three main assumptions of our framework are as follows:

1. The firm's profit from each sold unit is assumed in the form $p-Q$. Here p represents the firm's income from selling the item and $-Q$ represents the cost necessary to produce a product of quality Q. The use of a different quality–cost relationship does not significantly alter the framework's behavior as long as the basic condition of monotonicity (production cost grows with the product's quality) holds.
2. When offered a product of quality Q and price p, the probability that a given consumer accepts the offer and purchases the product – a so-called "acceptance" probability – is

$$P_A(Q, p) = (1 - p/p_{\mathrm{m}})(Q/p)^{\alpha}. \tag{1}$$

   Here the first term reflects that there is a maximal price p_{m} that the consumer can afford and the acceptance probability vanishes as p approaches to p_{m}. The second term reflects the consumer's evaluation of the product quality relative to the product price whereas α is a parameter characterizing how experienced is the consumer. Experienced consumers are able to assess the intrinsic product's quality and their acceptance probability is therefore substantial only when Q is close to p (they require value for their money) which corresponds to α being large. Little experienced consumers are characterized by low α and they are likely to accept also a product with bad Q/p ratio. Plots visualizing the behavior of the acceptance probability given by Eq. (1) are shown in Fig. 1. Note that Appendix presents a more fundamental derivation of the acceptance probability. Similarly, the value of p_{m} can vary between the consumers and thus reflect their diverse budget constraints. To limit the scope of our present work, we leave the case of heterogeneous p_{m} values and their impact on market equilibria for future study.
3. We assume that a consumer facing multiple offers first selects one of them and then decides whether to purchase it or not. It is natural to require that the probability to select a given product – a so-called "selection" probability – grows with the product's quality and decreases with the product's price. Since the acceptance probability $P_A(Q, p) = (1 - p/p_{\mathrm{m}})(Q/p)^{\alpha}$ has exactly these properties, we assume for simplicity that the probability of selecting a particular product is also proportional to $(1 - p/p_{\mathrm{m}})(Q/p)^{\alpha}$. The separation of consumer decision into a selection step and an acceptance step implies that even when several distinct products are available, at most one of them will be purchased by a given consumer.