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# Multifractal detrended cross-correlation analysis of carbon and crude oil markets

Xiaoyang Zhuang *, Yu Wei **, Bangzheng Zhang

*School of Economics & Management, Southwest Jiaotong University, First Section of Northern Second Ring Road, Chengdu, China*

## HIGHLIGHTS

- We confirm the presence of cross-correlations between carbon and crude oil markets both quantitatively and qualitatively.
- Based on the MF-DCCA approach, the nonlinear structure of the cross-correlations between carbon and crude oil markets is studied.
- The MF-DFA method is used to investigate the multifractal behaviors of carbon markets and crude oil markets respectively.

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ABSTRACT

The complex dynamics between carbon and crude oil markets have been an increasingly interesting area of research. In this paper, we try to take a fresh look at the cross-correlations between carbon and crude oil markets as well as their dynamic behavior employing multifractal detrended cross-correlation analysis. First, we find that the return series of carbon and crude oil markets are significantly cross-correlated. Second, we confirm the existence of multifractality for the return series of carbon and crude oil markets by the multifractal detrended fluctuation analysis. Third, based on the multifractal detrended cross-correlation analysis, we find the existence of power-law cross-correlations between carbon and crude oil markets. The cross-correlated behavior of small fluctuations is found to be more persistent than that of large fluctuations. At last, some relevant discussions and implications of the empirical results are presented.



## 1. Introduction

In the last few decades, the dynamics of financial markets have been of great interest. As far as we know, the previous studies have confirmed that financial markets are extremely complex and exhibit dynamics and non-linear properties, such as multifractality and long-range correlation features [1–6]. For instance, Alvarez-Ramirez et al. [7] found the dynamics and multifractal effect of the oil market by analyzing the auto-correlations of international crude oil prices. Gu et al. [8] studied the multifractal structure of WTI and Brent crude oil markets by employing the multifractal detrended cross-correlation analysis (MF-DCCA) method. Ho et al. [9] conducted the multifractal analysis on the Taiwan Stock index and the Hang Seng index. Kyungsik et al. [10] confirmed the multifractal properties of foreign exchange markets. Ioan et al. [11] investigated the multifractal behavior of Central and Eastern European foreign exchange rates.

In the previous works, various methods were developed to quantify the auto-correlation and cross-correlation behaviors of financial markets based on the monofractal and multifractal theory. Peng et al. [12] proposed the detrended fluctuation analysis (DFA) to explore the long-range auto-correlations of a non-stationary time series and widely used in financial

* Corresponding author.
** Corresponding author. Tel.: +86 13618001075.
*E-mail addresses:* zhuangxy19@163.com (X. Zhuang), weiyusy@126.com (Y. Wei).

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time series analysis [13,14]. Then, DFA was extended into two important methods: one is the multifractal detrended fluctuation analysis (MF-DFA) proposed by Kantelhardt et al. [15], which is a powerful tool to investigate the multifractality of the financial time series [16–18]; the other is the detrended cross-correlation analysis (DCCA) proposed by Podobnik and Stanley [19], which can be used to quantify the cross-correlations between two non-stationary financial time series. To examine the multifractal characteristics of two cross-correlated non-stationary time series, Zhou [20] proposed the multifractal detrended cross-correlation analysis (MF-DCCA) based on MF-DFA and DCCA. After that, MF-DCCA was widely used to investigate the cross-correlations in financial markets [21–25]. For example, Wang et al. [25] investigated the cross-correlations of West Texas Intermediate (WTI) crude oil spot and futures return series by means of MF-DCCA.

On connecting the environment energy problem and the economic development, the mechanism of carbon market becomes a hot topic. In addition, with the rapid growth of the carbon market, carbon price fluctuations are increasingly important for market participants. The price of carbon is classically driven by the balance between supply and demand, and by other factors related to the market structure and institutional policies. On the supply side, the number of allowances distributed is determined by Member-State through National Allocation Plans. On the demand side, the use of $CO_2$ allowances is a function of expected $CO_2$ emissions. In turn the level of emissions depends on a large number of factors, such as unexpected fluctuations in energy demand and energy prices. The demand for allowances can be affected by economic growth and financial markets as well, but that latter impact needs to be further assessed in the academic community.

In this paper, we try to take a fresh look at the cross-correlations between carbon and energy markets. Since fossil energy consumption has been considered as one of the main drivers of carbon prices, the price change of the energy market is a key to the international carbon market price formation mechanism. Mansanet-Bataller et al. [26] found that carbon prices are linked to the fossil fuel use. Alberola et al. [27] emphasized that the nature of the relationship between energy and carbon prices varies depending on the period under consideration. Convery and Redmond [28] found that an increase of fossil energy price will push the carbon price, and vice versa. Kanen [29] revealed that crude oil prices are the main thrust of the changes in natural gas prices; meanwhile fluctuations in natural gas prices will affect the tariff changes and ultimately impact the carbon price. Therefore, the analysis of cross-correlations between carbon and energy markets is essential and plays an important role in exploring the internal operation mechanism and forecasting the carbon price trend. In the paper, we choose crude oil markets as the representative of energy markets. Crude oil takes a big percentage in energy markets, and occupies a dominant position in the EU's energy consumption. In addition, the previous studies have already confirmed that the presence of correlation and multifractality in crude oil markets and also implies inefficiency [30–32].

The organization of this paper is as follows. Section 2 introduces methods employed in this study. Section 3 briefly described the data used in our work. Section 4 provides the detailed empirical results. Section 5 presents some discussion. Then, the last section concludes.

## 2. Methodology

There are two time series $\{x(i)\}$ and $\{y(i)\}$, $i = 1, 2, \ldots, N$, where $N$ is the length of the series. The MF-DCCA method can be summarized as follows:

First, calculate the profile

$$X(i) = \sum_{k=1}^{i} (x(k) - \bar{x}), \qquad Y(i) = \sum_{k=1}^{i} (y(k) - \bar{y}), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\bar{x}$ and $\bar{y}$ denote the averaging over the two whole time series $x(i)$ and $y(i)$.

Second, divide the two profiles $X(i)$ and $Y(i)$ into $N_s = \mathrm{int}(N/s)$ non-overlapping segments of equal length $s$. Since the length $N$ of the series is often not a multiple of the given time scale $s$, a short part at the end of each profile may remain. In order not to discard this part of the series, the same procedure is repeated starting from the opposite end of each profile. Thereby, $2N_s$ segments are obtained together. Then, estimate the local trends for each of the $2N_s$ segments by means of the $m$th order polynomial fit. Then the detrended covariance is determined by

$$F^2(s, \lambda) = \frac{1}{s} \sum_{j=1}^{s} \left| X_{(\lambda-1)s+j}(j) - \tilde{X}_\lambda(j) \right| \left| Y_{(\lambda-1)s+j}(j) - \tilde{Y}_\lambda(j) \right| \tag{2}$$

for each segment $\lambda$, $\lambda = 1, 2, \ldots, N_s$ and

$$F^2(s, \lambda) = \frac{1}{s} \sum_{j=1}^{s} \left| X_{N-(\lambda-N_s)s+j}(j) - \tilde{X}_\lambda(j) \right| \left| Y_{N-(\lambda-N_s)s+j}(j) - \tilde{Y}_\lambda(j) \right| \tag{3}$$

for each segment $\lambda$, $\lambda = N_s + 1, N_s + 2, \ldots, 2N_s$. Here, $\tilde{X}_\lambda(j)$ and $\tilde{Y}_\lambda(j)$ denote the fitting polynomial with order $m$ in segment $\lambda$.

Then average over all segments to obtain the $q$th-order fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\lambda=1}^{2N_s} \left[ F^2(s, \lambda) \right]^{q/2} \right\}^{1/q} \tag{4}$$