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Epidemic spreading on weighted adaptive networks

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HIGHLIGHTS

- We propose a weight adaptive network model on SIS dynamics.
- The weight of links can be transferred among links.
- The weight adaption process could aggravate the prevalence of an epidemic.
- The weight adaption process is further applied to link-removal immunization strategy.
- The weight adaption process may weaken the efficiency of the strategy.

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ABSTRACT

Considering the fact that the contact strengths among people are diverse both in the duration time and the distance, in this paper, we study the epidemic dynamics with susceptible–infective–susceptible (SIS) model on a weighted adaptive network to emphasize this contact feature. In this model, the weight of a link denotes the contact strength between two individuals connected by this link, and each susceptible individual may adaptively transfer the weight from a link to another. We find that this weight adaption process could significantly aggravate the prevalence of an epidemic. Moreover, we examine the effectiveness of the link-removal strategy with our model, and the results show that the weight adaption process may weaken the efficiency of the strategy. The theoretical analysis is supported by the simulation results.

1. Introduction

Human relations and activities are not only the driving force of various social events, but are also affected by many related dynamical processes. The influence of social relations on the epidemic spreading is one of the important concerns. Many studies of epidemic spreading on complex networks have focused on how the network topology affect the properties of the epidemic dynamics, such as the infected regime [1–3], the outbreak threshold [4–8], the spreading velocity [9–11]. Several works have discussed epidemics on networks whose structures change independently to the epidemic states of the nodes [12–17].

However, real-world social networks are formed by entities whose interactions change dynamically. Considering the fact that when a society is under the threaten of the prevalence of an infectious disease, individuals may adjust their relations in response to their infection states, Gross et al. introduced an adaptive network model based on an endemic susceptible–infective–susceptible (SIS) model [18,19]. In this model, the structure of the network changes all the time according to the nodal infection states and in turn affects epidemic dynamics. Specifically, the susceptible nodes may rewire their connections in order to avoid the contact with their infected neighbors. Interestingly, a bi-stability phenomenon is observed in their

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model. Later, the stochastic impact on the adaptive networks is discussed on susceptible–infective–recovered–susceptible (SIRS) model [20], and the case that the links between susceptible nodes and their infected neighbors are removed permanently is considered, which leads to a quick extinction of the epidemic spreading [21]. Recently, methods for accurate prediction of the evolution of adaptive networks are developed [22,23]. Generalized models are proposed where adaptive dynamics based on local information are considered [24,25].

When an epidemic outbreaks, governments may perform diverse intervention methods to control its diffusion. However, it has been experienced that some nonpharmaceutical measures are not effective at mitigating the disease outbreaks. For example, it is reported that in Singapore more than 10 preschools closed for 10 days due to outbreaks of hand, foot and mouth disease (HFMD) among the pupils, but the number of cases still remained close to record levels when schools reopened [26]. Hence, more works are deserved to understand why sometimes the isolation strategy is not so effective. We notice that in the real world, the contact strength is variable between different pairs of acquaintances rather than being identical [27–29]. Moreover, once the isolation strategy is performed when an epidemic is prevalent, the contact strength between infected ones and susceptible ones is reduced, the contact frequency between the susceptible and others may be strengthened. Therefore, considering these two factors, in this paper we propose a weighted adaptive network model, where a weight assigned to a link describes the contact strength between the two nodes connected by the link. The weight here could be regarded as interaction time or the proximity of encounters [28–34]. In our model, the susceptible nodes may reduce the weight of the connections going to infected neighbors, and then they may transfer this weight to other connections going to his susceptible neighbors. We find that, compared with un-weighted model, this weight transferring process could significantly aggravate the damage of an epidemic. Besides, we examine a simple, but generic, epidemic control intervention by randomly removing a fraction of links and find the weighted adaption process could significantly weaken the effectiveness of the strategy.

In Section 2, we introduce the weighted adaptive network model. In Section 3, we give the theoretical formulism of our model. In Section 4, we present the numerical simulations and discuss our observations. The effectiveness of the isolation intervention strategy is examined on our model in Section 5. Finally, we concludes this work in Section 6.

2. Model

Suppose initially a network contains N nodes and M links, and the weight assigned to each links is set to be unit. At each time step, there are two dynamical processes. The first one is the epidemic spreading process. We employ the SIS model to describe this process [4], where all the nodes can be in two states: susceptible (S) and infected (I). We use S and I to represent the number of nodes in the two groups respectively. Hence, we have $N = S + I$. Initially, a small number of nodes are set to be infected as the seed of the infection, while all the other nodes are susceptible. In order to measure the infection rate of a weighted SI link, which connects a susceptible node and an infected one, we assume that the probability of a susceptible node getting infected through an SI link with unit weight, denoted as w_1, is p_1. Thus for any SI link with weight w, the infection rate is $p_w = 1 - (1 - p_1)^{w/w_1}$. When p_1 is small, it yields $p_w \simeq \frac{w}{w_1} p_1$. Without losing generality, we set unit weighted $w_1 = 1$ and let $p_1 = p$, then we have $p_w = pw$. The weight of a link is symmetry, which means the weight $w(i, j)$ of a link connecting node i and j has $w(i, j) = w(j, i)$. If a susceptible node i has n infected neighbors, assigned $j_1, j_2, \ldots, j_n$, the probability for node i being infected is $[1 - \prod_{k=1}^{n}(1 - pw(i, j_k))]$. When p is small, the probability can be approximated to $\sum_{k=1}^{n} pw(i, j_k)$. Besides, each infected node can recover to be susceptible with a probability γ.

The second process is the weight transferring process, which is composed of two steps: (i) *Weight Reducing*: Each susceptible node might reduce the weight of each link connecting to an infected neighbor by a fixed amount δ in a rate ν. When the weight of a link drops to zero, this link is removed permanently. (ii) *Weight Adding*: Once the weight of an SI link has been reduced by δ, the susceptible node will transfer this weight δ to a link randomly chosen from all the SS links connecting to his susceptible neighbors with probability q (see Fig. 1). If this node has no SS link attached it, this step is skipped, which induces a decrease of the total weight of the network. The process that only susceptible neighbors can be selected for weight transferring is based on two considerations: first, it is easier for a person to contact his acquaintances, such as families and friends, than to build a relation with unfamiliar ones. Second, it is more difficult to identify the healthy condition of an unfamiliar person than a familiar one. Therefore, it is reasonable to contact a familiar person than an unfamiliar one when a disease is in prevalence. Since all the susceptible nodes only choose their neighbors to transfer the weight, the number of links will not increase in the whole process. When $q = 0$, the model turns back to the one in Ref. [21], in which the broken links between susceptible nodes and infected ones are permanently removed. In this case the weight transferring process does not happen actually, the weight of all the links in the network keeps unit. For the simplicity of discussion, we call the this case as un-weighted adaptive model. Furthermore, in this paper, we set $\delta = 1$. We have tried other values, such as $\delta = 1/2$ and $1/3$, and the main results also hold.

3. Theoretical formulism

We first develop a set of differential equations to describe the dynamics of our model. We use $L_{XY}(w)$ to denote the number of links with weight w connecting a node in state X to a node in state Y where X and Y can either be S state or I