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Selection of minimal length of line in recurrence quantification analysis

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HIGHLIGHTS

- It was revealed that *Det* of Lorenz time series has normal probability distribution.
- The variance of *Det* versus $l_{\min}$ has a convex parabolic shape.
- The results prove that the value of $l_{\min} = 2$ is the best selection.
- The optimum $l_{\min}$ improves *Det* to distinguish between different dynamics of a system.

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ABSTRACT

A qualitative analysis along with mathematical description was made on the selection of the optimal minimal length of line, $l_{\min}$, a crucial parameter in the recurrence quantification analysis (RQA). The optimum minimal length of line is defined as a value that enhances the capability of RQA variables (determinism, in this paper) to distinguish between different dynamic states of a system. It was shown that the determinism of the Lorenz time series has a normal distribution. The results indicated that the lowest possible value of the minimal length of line (i.e., $l_{\min} = 2$) is the best choice. This value provides the highest differentiation for determinism of the time series obtained from different dynamic states of the Lorenz system. The applicability of the results was verified by examining determinism for monitoring the fluidization hydrodynamics.

1. Introduction

Recurrence quantification analysis (RQA) has found special attention in the study of dynamic systems, in particular those that encompass complex dynamic features. Real world dynamic systems, like weather, the human body, stock price and industrial and engineering equipments, typically involve complex chaotic nonlinear behavior. These systems are governed by complex nonlinear dynamic relationships and are modeled by mathematical tools such as differential equations. Most of these systems have chaotic dynamics (i.e., they are highly sensitive to the initial condition) and their dynamic state cannot be predicted over a long time. Hence, real world dynamic systems usually are studied experimentally through time series evaluation of the measured signals of the underlying system.

There are various statistical tools to study and characterize the dynamic features of time series [1]. Recurrence plot (RP) and RQA are unique and influential statistical tools for studying and characterizing of time series owing to applicability to both non-stationary and stationary time series [2,3]. For construction of the RP, any two points of underlying time series are compared to find whether their difference is smaller than the radius threshold or not. If the difference is less than the

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Notations

Det	Determinism
$l_{\min}$	Minimal length of diagonal lines
N	Number of time series points
$P(l)$	Number of diagonal lines of length *l*
r	Radius threshold

Greek letters

P	Reduced Rayleigh number
B	Geometric factor
Σ	Prandtl number
σ^2	Variance

Abbreviations

RP	Recurrence plot
RQA	Recurrence quantification analysis

radius threshold, that point would be considered as a recurrence state [2]. While attractor reconstruction in the phase space is obligatory in a conventional chaotic based statistical analysis, March et al. [4] showed that embedding of time series in construction of RP is not mandatory. In addition, the RQA can be helpful for short-term time series as well as long-term time series in the dynamical system studies [2,3,5]. Numerous investigations have been performed using RQA in various fields, such as physiology, neuroscience, economics, engineering, physics, chemistry, medicine and earth science [2,3,6–9]. In most of these investigations, a measurable property of a real world dynamic system was collected as a time or spatial series and analyzed by RQA variables.

Even though there are various methods for analyzing time series [1], the RQA has unique properties. In many industrial situations, it is intended to prevent the deviation of a system from a predefined optimum operating condition. This task is usually done through monitoring different variables of the system, such as temperature, pressure and composition. However, this practice is not always straightforward because in some cases, in particular in nonlinear systems, unwanted phenomena which divert the system do not always have an obvious effect on the immediate value of the measured variables. An example for this problem is agglomeration in fluidized bed polymerization reactors [10]. In these cases, time series of a characteristic variable of the system can provide useful information about its dynamics [11]. In order to determine if the system remains in the optimum condition, the measured time series should be analyzed by statistical methods to derive the information related to the system's dynamic. The phenomena that drive the system out of its previous state usually cause the time series to become non-stationary. However, most nonlinear analyzing methods cannot be applied to non-stationary time series. In addition, reconstruction of an attractor in a nonlinear method requires a time series with a large length. Nevertheless, the RQA not only can be applied for non-stationary time series but also can applied for short-term time series. These two properties make the RQA a special tool for monitoring nonlinear dynamics.

The RQA consists of a set of different variables such as recurrence rate, determinism, laminarity, trend, entropy, etc. [2,3] that quantify constructing structures of a recurrence plot such as single points, horizontal (vertical) lines and diagonal lines, [2,3,12]. Diagonal lines reflect predictability of the time series, the main feature of a chaotic dynamic. Determinism quantifies diagonal lines and is one of the most applicable variables of the RQA. Determinism is mathematically defined as [2,3]:

$$Det = \frac{\sum_{l=l_{\min}}^{N} (l \times P(l))}{\sum_{l=1}^{N} (l \times P(l))} \tag{1}$$

where $P(l)$ is the number of lines which are of length *l*. Also, $\sum (l \times P(l))$ represents the number of recurrence points forming diagonal lines. For $l_{\min} = 1$, *Det* is 100%. In that case, all points are considered as diagonal lines and no differentiation can be made between different dynamic states. Therefore, the minimal length of the line should be greater than one so the determinism can recognize these behaviors precisely.

Iwanski and Bradley [13] showed that determinism is insensible to the embedding dimension. As previously mentioned, determinism (and other RQA variables) can be applied to both non-stationary and short-term time series. This is also the case for statistical moments (mean, variance, etc.). However, a great advantage of the RQA is that its parameters do not eliminate the information contained in the sequence of points of the time series. Such a property makes the RQA a useful tool for monitoring the systems which should be monitored through time series evaluation of its characteristic variables.