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Is the wealth of the world's billionaires not Paretian?

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HIGHLIGHTS

- The Paretian behavior of the wealth of the world's billionaires has been tested.
- Previous tests rejected Paretian behavior.
- Previous tests did not account for measurement errors.
- Tests that account for measurement errors can fail to reject Paretian behavior.

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ABSTRACT

According to previous studies that applied a popular goodness-of-fit test, the wealth of the world's billionaires does not follow a Pareto distribution. The test applied by those studies assumes that wealth is measured without error, yet, if different sources of data on the wealthiest people in the world are compared, then wealth appears to be measured with error. This paper shows that the conclusions drawn from the goodness-of-fit test can change when the test is modified to account for measurement errors.

1. Introduction

Since at least a study that appeared in this journal over a decade ago [1], there have been a number of studies on whether wealth follows a Pareto or power-law distribution among the wealthiest people in the United States [2,3], other countries [4,5], or the entire world [6,7]. While earlier studies relied on little more than visual inspection, more-recent studies have relied on formal goodness-of-fit tests. Recently, a study in this journal [8] and another similar study [9] applied the popular Kolmogorov–Smirnov (K–S) goodness-of-fit test in order to test whether the wealth of the world's billionaires follows a Pareto distribution. Like earlier studies on the same subject [6,7], the source of data used by those studies was an annual list of the world's billionaires published by *Forbes*. Applying the K–S test to the Forbes list, the studies concluded that the distribution of wealth among the world's billionaires is such a poor fit to a Pareto distribution that such a fit should almost never be observed if their wealth actually follows a Pareto distribution. The same conclusion has been drawn about the distribution of wealth among the wealthiest people in specific countries by using similar lists and the same test [2,4,9].

Lists of named wealth-holders like the Forbes list are the best source of data on the world's wealthiest people, given that other sources of data like surveys and tax records often fail to capture those people [6], but the Forbes list suffers from rounding errors and it may also suffer from other measurement errors. Comparing the list published by *Forbes* to similar lists published by *Bloomberg* and the *Hurun Report*, the same people are worth different amounts according to the different lists. Such discrepancies suggest that all of the lists, including the Forbes list, may suffer from other measurement errors besides rounding errors. Although the Forbes list suffers from at least rounding errors, the K–S test as it was applied by

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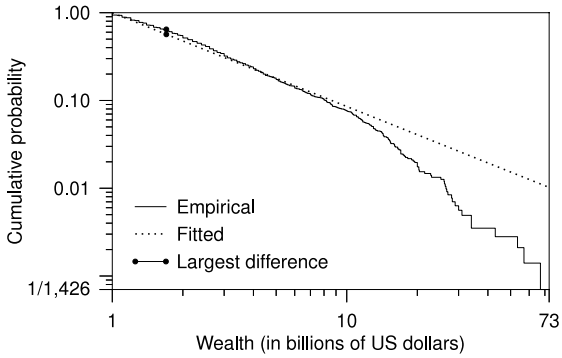

Fig. 1. Empirical CCDF for the wealth of the people on the 2013 Forbes list, the CCDF for a Pareto distribution that was fit to the Forbes list, and the largest absolute difference between the empirical and fitted CCDFs.

previous studies assumes that wealth is measured without any error. This paper shows that, after accounting for rounding errors, and after accounting for other measurement errors by using the discrepancies between the different lists, the conclusions of previous studies can be overturned for at least one point in time at which the different lists can be compared. Conclusions about whether wealth follows a Pareto distribution may therefore depend on assumptions about whether wealth is measured with error.

The rest of this paper is organized as follows. Section 2 discusses the manner in which previous studies applied the K–S test to the Forbes list. Section 3 then argues that the Forbes list suffers from measurement errors and that the K–S test should be applied in a manner that accounts for those errors. The same section also shows that, when the test is applied in such a manner, its conclusions can change. The last section concludes.

2. A goodness-of-fit test

Over a century ago, Vilfredo Pareto used income-tax data to discover that, if the number of people with income greater than a given level of income was plotted against income on a double logarithmic scale, then the plot looked approximately like a straight line, at least at the highest levels of income. What is now called a "Pareto" or "power-law" distribution has the complementary cumulative distribution function (CCDF)

$$P(X > x) = (b/x)^a$$

for $x \geqslant b$, where $b > 0$ is a lower-bound parameter and $a > 0$ is a shape parameter [10]. That function is simply a formalization of the distribution that Pareto discovered for the distribution of income. If the function was plotted on a double logarithmic scale, then it would look exactly like a straight line with a slope equal to the (negative of the) shape parameter.

Ever since Pareto's discovery, scholars have debated whether income [11,12] and other economic variables including financial returns [13,14] and financial ratios [15] follow a Pareto distribution. Scholars have also debated whether wealth follows a Pareto distribution. The two studies mentioned above [8,9] recently entered into the ongoing debate over whether wealth follows a Pareto distribution among the world's billionaires. The above-mentioned studies used the same source of data used by earlier studies (again, a list of the world's billionaires published by *Forbes* on an annual basis), but, unlike earlier studies that relied on little more than visual inspection, those studies applied the K–S goodness-of-fit test.[1]

The K–S test can be illustrated by applying it to the Forbes list from 2013, which is the most-recent year of that list, as of writing [16]. Wealth is a stock variable, so it should be measured at a point in time. The Forbes list from 2013 was supposed to be a snapshot of wealth on February 14th of that year. Part of Fig. 1 shows the empirical CCDF for the wealth of the world's billionaires on that day of the year, according to the Forbes list. As seen in the figure: There were apparently 1426 people in the world who were worth at least one billion US dollars on that day of the year, and the wealthiest one of them was apparently worth 73 billion dollars.

Another part of the same figure shows the CCDF for a Pareto distribution that was fit to the Forbes list by maximum likelihood [10].[2] The largest absolute difference between the empirical and fitted CCDFs is also shown as part of that figure. The largest difference is about eight percent. That difference occurs once at a wealth of about 1.7 billion dollars. Out of the 1426 people on the Forbes list, 921 or about 65% of them were worth at least 1.7 billion dollars, but the Pareto distribution that was fit to the Forbes list suggests that only about 808 or 57% of them should have been worth at least that much.

[1] As noted below, one of the studies [9] also used the same test to see whether the distribution of wealth among wealthier groups than the world's billionaires was a better fit to a Pareto distribution, while the other study [8] used other goodness-of-fit tests; but, again, both studies used the K–S test to see whether the distribution of wealth among the world's billionaires was a good fit to a Pareto distribution.

[2] The two studies mentioned above [8,9] also used maximum likelihood, although neither study appears to have adjusted for the maximum likelihood estimator's bias. We used the adjustments suggested by [10].