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Ultra-relativistic nonthermal power-law ensembles: Cosmic-ray electrons and positron fraction

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HIGHLIGHTS

- Relativistic power-law densities admit a stable and extensive entropy functional.
- Positive heat capacities and compressibility ensure their thermodynamic stability.
- Quantization of nonthermal relativistic power-law ensembles in Fermi statistics.
- Spectral fitting of nonthermal plasmas with ultra-relativistic power-law densities.
- Entropy, temperature and chemical potential of the cosmic-ray electron/positron flux.

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ABSTRACT

Thermodynamically stable ultra-relativistic power-law distributions are employed to model the recently measured cosmic-ray electron flux and the positron fraction. The probability density of power-law ensembles in phase space is derived, as well as an extensive entropy functional. The phase–space measure is transformed into a spectral number density, parameterized with the Lorentz factor of the charges and quantized in Fermi statistics. Relativistic power-law ensembles admit positive heat capacities and compressibilities ensuring mechanical stability as well as positive root mean squares quantifying thermodynamic fluctuations. The wideband spectral fitting of dilute nonthermal electron–positron plasmas with ultra-relativistic power-law densities is explained.

1. Introduction

We study nonthermal power-law ensembles capable of reproducing the flux of ultra-relativistic particles [1–5] such as cosmic-ray electrons and positrons. In particular, we derive power-law distributions from empirical spectral fits which are thermodynamically and mechanically stable, admitting positive heat capacities and compressibilities. The entropy of power-law ensembles is shown to be an extensive variable, and the positivity of the root mean squares of thermodynamic variables is explicitly demonstrated, ensuring a negative second entropy differential $d^2S \leq 0$ subject to the extremal condition $dS = 0$.

The spectral number density of ultra-relativistic power-law ensembles admits a convenient parameterization by the Lorentz factor of the particles. We derive the classical number density from the phase–space probability measure, which is taken as the starting point for the quantization of classical power-law ensembles in Fermi statistics. We also briefly

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outline the bosonic quantization of power-law densities as well as the fermionic counterpart to electromagnetic gray-body radiation, with Lorentz factors defined by group velocity in a dispersive spacetime. Hybrid quantum distributions interpolating between Fermi and Bose statistics are derived.

In Section 2, we give an overview of the power-law densities studied, including their partition function, the basic thermodynamic variables, their variances, the equations of state, and show thermodynamic stability. In Section 3, we study classical power-law densities in phase space, their probability density and entropy functional, fluctuations of internal energy, particle number and entropy, as well as the quantization of nonthermal power-law ensembles.

In Section 4, we discuss wideband spectral fitting with ultra-relativistic power-law densities. The power-law densities studied here are aimed at this purpose, being thermodynamically stable empirical distributions sufficiently general to reproduce the multiple spectral peaks of broadband spectra. We show how to extract temperature, fugacity and chemical potential, as well as the power-law amplitudes and exponents from the measured particle flux. We also discuss energy scales and conditions for the applicability of the classical limit of the distributions.

In Section 5, we perform a spectral fit to the cosmic-ray electron flux, employing an ultra-relativistic classical power-law density. We use GeV data sets collected with the PAMELA and Fermi satellites [6–8], extended into the low TeV region by an atmospheric Cherenkov array [9–11]. Based on the electronic number density (obtained from the spectral fit) and the recently measured positron fraction, we calculate the spectral number density of the cosmic-ray positron flux in the 1 GeV–3 TeV interval. Finally we estimate the specific energy and number densities, as well as temperature and entropy production of ultra-relativistic cosmic-ray electrons/positrons, which constitute a nonthermal two-component plasma, dilute and at high temperature. In Section 6, we present our conclusions.

2. Fermionic power-law distributions

2.1. General outline: spectral number density, partition function, entropy

Fermionic power-law ensembles are defined by the distributions (number densities)

$$\mathrm{d}\rho_{\mathrm{F}}(\gamma) = \frac{m^3 s}{2\pi^2(\hbar c)^3}\frac{\sqrt{\gamma^2-1}\,\gamma\,\mathrm{d}\gamma}{1+G(\gamma)\,\mathrm{e}^{\beta\gamma+\alpha}}, \tag{2.1}$$

parameterized by the Lorentz factor γ. The positive spectral function $G(\gamma)$ in the denominator is normalized as $G(1) = 1$ and is to be determined from a spectral fit. The mass parameter m is a shortcut for the rest energy mc^2, so that $E = m\gamma$ is the particle energy, and s is the spin multiplicity. The dimensionless temperature parameter is $\beta = m/(k_{\mathrm{B}}T)$, with Boltzmann constant k_{B}. The dimensionless real fugacity parameter α is related to the chemical potential μ by $\alpha = -\beta\mu/m$, and $z = \mathrm{e}^{-\alpha}$ is the fugacity. We specify $G(\gamma)$ as a power-law ratio,

$$G(\gamma) = \gamma^{\delta}\frac{g(\gamma)}{g(1)}, \qquad g(\gamma) = \frac{1+(\gamma/a_1)^{\sigma_1}+\cdots+(\gamma/a_n)^{\sigma_n}}{1+(\gamma/b_1)^{\rho_1}+\cdots+(\gamma/b_k)^{\rho_k}}, \tag{2.2}$$

where δ is a real power-law exponent. The amplitudes a_i, b_i defining $g(\gamma)$ are positive, and so are the exponents, $0 < \sigma_1 < \cdots < \sigma_n$ and $0 < \rho_1 < \cdots < \rho_k$. Empirical spectral functions of this type suitable to model multiple spectral peaks in wideband spectra have been suggested in Refs. [1–3], a specific choice of the ratio $g(\gamma)$ is worked out in Sections 4 and 5. Fermionic power-law ensembles with $g(\gamma) = 1$ have been studied in Refs. [12–16], and thermodynamic stability of the ratios (2.2) is demonstrated in Section 2.2.

The particle number in a volume V is

$$N = -\frac{\partial}{\partial\alpha}\log Z_{\mathrm{F}} = V\int_{\gamma_{\mathrm{cut}}}^{\infty}\mathrm{d}\rho_{\mathrm{F}}(\gamma), \tag{2.3}$$

referring to particles with energies exceeding $E_{\mathrm{cut}} = m\gamma_{\mathrm{cut}}$, where $\gamma_{\mathrm{cut}} \geq 1$ is the cutoff Lorentz factor. The partition function Z_{F} is stated in (2.6). The ultra-relativistic limit of $\mathrm{d}\rho_{\mathrm{F}}(\gamma)$ is obtained by replacing $\sqrt{\gamma^2-1} \to \gamma$ in (2.1), which applies if $\gamma_{\mathrm{cut}} \gg 1$. The thermal Fermi–Dirac distribution is recovered with $\delta = 0$, $g(\gamma) = 1$ and $\gamma_{\mathrm{cut}} = 1$ as the lower integration boundary of the partition function. Here, we study nonthermal averages with real power-law index δ and empirical spectral function $g(\gamma)$ extracted from spectral fits of measured flux densities.

The spectral number density (2.1) is based on the dispersion relation $p = (m/c)\sqrt{\gamma^2-1}$. The integration in momentum space is parameterized as $\mathrm{d}^3p = 4\pi p^2 p'(\gamma)\,\mathrm{d}\gamma$, so that the particle density (2.1) is compiled as

$$\mathrm{d}\rho_{\mathrm{F}} = \frac{s}{(2\pi\hbar)^3}\frac{\mathrm{d}^3p}{1+G(\gamma)\,\mathrm{e}^{\beta\gamma+\alpha}}, \qquad \mathrm{d}^3p = \frac{4\pi m^3}{c^3}\sqrt{\gamma^2-1}\,\gamma\,\mathrm{d}\gamma. \tag{2.4}$$

Remark. We may consider a more general dispersion relation, $p = (m/c)\hat{\mu}\sqrt{\hat{\varepsilon}^2\gamma^2-1}$, with energy-dependent dimensionless permeabilities $\hat{\varepsilon}(\gamma)$ and $\hat{\mu}(\gamma)$ resulting in a dispersive power-law density, a massive counterpart to photonic gray-body radiation. The integration in (2.3) is then over intervals in which the radicand of $p(\gamma)$ is positive, and the normalization is done with the smallest possible Lorentz factor, $g(\gamma_{\mathrm{min}})$ instead of $g(1)$. We still have $\gamma = E/m$, and the Lorentz factor γ can be parameterized by group velocity via $m/\upsilon = \mathrm{d}p/\mathrm{d}\gamma$. Here, we use vacuum permeabilities, $\hat{\varepsilon} = \hat{\mu} = 1$.