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Directed clustering coefficient as a measure of systemic risk in complex banking networks☆

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HIGHLIGHTS

- We show that directed clustering coefficients can be interpreted as measures of risk taking in financial networks.
- We discuss the interpretation of these coefficients using a unique dataset for the Brazilian interbank market.
- We show empirical results for the Brazilian economy.

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ABSTRACT

Recent literature has focused on the study of systemic risk in complex networks. It is clear now, after the crisis of 2008, that the aggregate behavior of the interaction among agents is not straightforward and it is very difficult to predict. Contributing to this debate, this paper shows that the directed clustering coefficient may be used as a measure of systemic risk in complex networks. Furthermore, using data from the Brazilian interbank network, we show that the directed clustering coefficient is negatively correlated with domestic interest rates.

1. Introduction

Recent literature has focused on the issue of modeling, measuring and avoiding systemic risk in complex networks [1]. The literature is divided into general approaches such as [2–7] and in techniques that deal with specific kinds of networks, such as technological networks [8–11], social and biological networks [12] and financial and economic networks [13–16]. In the particular case of financial and economic systems, based on the events that took place in the crisis of 2008, it is clear that the aggregate behavior of the interaction among agents is not straightforward and it is very difficult to predict [17]. Furthermore, the challenge of understanding aggregate behavior of economic and financial systems require tools belonging to the field of econometrics of times series, complex systems, game theory and agent-based models [18].

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Banking lending networks are one of the most important financial systems that are subjected to systemic risk. In fact, small shocks constrained only to a few banks can be spread by contagion and affect the entire system. Allen and Gale [19] show that in a banking system with a homogeneous topology, the possibility of financial contagion depends strongly on the completeness of the structure of the system. It is worth mentioning that due to the development of the theory of complex networks [20–22], it has been possible to improve our knowledge on banking networks. Now, knowing that banking networks have complex structure and dynamics [23–32], models of heterogeneous banking networks have replaced the homogeneous ones. For instance, exploring the interconnections among banks in the Italian overnight market, Iori et al. [33] have investigated potential implications of the current institutional system on banking stability. Nier et al. [34] simulated banking systems showing how systemic risk depends on their structures.

The main contribution of this paper is to develop a new interpretation of clustering coefficients as systemic risk measures. Different clustering coefficients are evaluated and discussed in the paper and their appropriate interpretation follows. In this context, this paper shows that the directed clustering coefficient [35] may be used as a measure of systemic risk in complex networks. We also correlate these clustering coefficients with interest rates to show that the interbank topology changes with the macroeconomic environment. In particular, exploring data from the Brazilian interbank network, we show that the way that banks make clusters of lending relationships affects the systemic risk.

The remainder of this paper is structured in the following way. Section 2 revisits the measure known as the directed clustering coefficient. Section 3 describes the data of the Brazilian interbank market used in this paper. Section 4 presents the main results of the paper. In particular, we show that the directed clustering coefficient is negatively correlated with interest rate changes and it varies strongly among banks. Finally, Section 5 presents the main conclusions of this work.

2. Clustering coefficients for directed networks

In Ref. [35], the standard clustering coefficient [36], used for unweighted and undirected networks, was generalized for *binary directed networks* and *weighted directed networks*. Consider the following notation. Let A and W be respectively the directed adjacency matrix of the network and directed matrix of weights that represents the network. Let also d_i^{in}, d_i^{out} and $d_i^{tot} = d_i^{in} + d_i^{out}$ be respectively the in-degree of node i, the out-degree of node i, and the total degree of node i. Furthermore, let $d^{\leftrightarrow} = \sum_{j \neq i} a_{ij} a_{ji} = A_{ii}^2$. In binary directed networks, the clustering coefficient of node i for a binary network may be defined as the ratio between all the possible triangles formed by i and the number of all possible triangles that could be formed

$$C_i^D(A) = \frac{(A + A^T)_{ii}^3}{2[d_i^{tot}(d_i^{tot} - 1) - 2d_i^{\leftrightarrow}]}. \tag{1}$$

This clustering coefficient defined for the unweighted case can be easily extended to the weighted case by replacing the number of directed triangles formed with its weighted counterpart

$$\tilde{C}_i^D(W) = \frac{[\hat{W} + (\hat{W}^T)]_{ii}^3}{2[d_i^{tot}(d_i^{tot} - 1) - 2d_i^{\leftrightarrow}]} \tag{2}$$

where $\hat{W} = W^{[\frac{1}{3}]} = [w_{ij}^{\frac{1}{3}}]$.

However, as pointed in Ref. [35], these two definitions (1) and (2) are not enough to characterize the richness of patterns that take place in a complex directed network. In fact, Eqs. (1) and (2) treat all the possible triangles as if they were the same. However, in directed graphs, edges that point to different directions should be interpreted differently. Therefore, four more definitions are necessary, which are represented in Fig. 1:

(a) cycle, when there is a cyclical relation among i and its neighbors. In this case, the associated clustering coefficient for the binary case is

$$C_i^{cyc} = \frac{(A)_{ii}^3}{d_i^{in} d_i^{out} - d_i^{\leftrightarrow}} \tag{3}$$

and for the weighted case is given by

$$\tilde{C}_i^{cyc} = \frac{(\hat{W})_{ii}^3}{d_i^{in} d_i^{out} - d_i^{\leftrightarrow}}. \tag{4}$$

(b) Middleman, when one of the neighbors of node i holds two outward edges and the other holds two inward edges. In this case, the associated clustering coefficient for the binary case is

$$C_i^{mid} = \frac{(AA^T A)_{ii}}{d_i^{in} d_i^{out} - d_i^{\leftrightarrow}} \tag{5}$$

and for the weighted case is given by

$$\tilde{C}_i^{mid} = \frac{(\hat{W}\hat{W}^T\hat{W})_{ii}}{d_i^{in} d_i^{out} - d_i^{\leftrightarrow}}. \tag{6}$$