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Discrete opinion models as a limit case of the CODA model

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HIGHLIGHTS

- We introduce reasoning about how an agent influences its neighbors in the CODA model.
- We show the discrete models can be obtained as a limit of this model.
- The dynamics of the system out of the limit case is studied, as a function of the agent's own influence.

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ABSTRACT

Opinion Dynamics models can be, for most of them, divided between discrete and continuous. They are used in different circumstances and the relationship between them is not clear. Here we will explore the relationship between a model where choices are discrete but opinions are a continuous function (the Continuous Opinions and Discrete Actions, CODA, model) and traditional discrete models. I will show that, when CODA is altered to include reasoning about the influence one agent can have on its own neighbors, agreement and disagreement no longer have the same importance. The limit when an agent considers itself to be more and more influent will be studied and we will see that one recovers discrete dynamics, like those of the voter model in that limit.

1. Introduction

Opinion Dynamics [1–7] modeling lacks a clear theoretical basis and connections between different models. Unifying proposals exists for discrete opinion models [8,9], but they do not include continuous opinions. Understanding how continuous models relate to the discrete ones, if at all, can help us move towards a better understanding of how to describe real social systems.

Here, I present a variation of the Continuous Opinions and Discrete Actions (CODA) model [10,11] where an agent considers its own influence on its neighbors. The purpose is both to present the model and to discuss how more traditional models relate to the framework and how they can be seen as approximations or limit cases. In Section 2, the relation between discrete spin models, where no probability or strength of opinion exists, and the proposed framework, is explained and I demonstrate for the first time how discrete models can be understood as a limit case of this framework, when agents consider their own influence on their neighbors. Using that demonstration as basis, the original model where each agent considers its own influence on others is introduced in Section 3 and we find out that, in the limit of very strong influence, spin dynamics are recovered. We will understand how the model allows any finite system to reach consensus and see that, for finite systems of any size, there is always a range of parameters where the model presented here is identical in results to a spin system. We will also see that, outside the limit, the model has interesting properties about the amount of extremism in the system.

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2. Discrete opinions and the CODA model

In the Continuous Opinions and Discrete Actions (CODA) model [10,11], each agent i is trying to decide between two conflicting options. This can be represented by a choice between two alternatives, A and B. This can be represented as a discrete variable x with only two possible values, assumed here to be ± 1. In order to decide which option is better, the agent relies on a subjective probabilistic evaluation of the problem. That is, agent i assigns a probability p_i to the statement that A (or $+1$) is the best choice. Therefore, the probability that B (or -1) is the best choice will be $1 - p_i$, according to i. The communication between agents here will only involve stating which choice is preferred by the agent and not the probability it assigns to that choice. That is, what is observed is the spin s_i, the sign of the choice, given by $s_i = \text{sign}(p_i - 0.5)$. Finally, when agent i observes agent j, it considers that, if A is the best choice, there is a chance a that j will pick it. In probability terms, we have the likelihood $p(s_j = +1|x = +1) = a > 0.5$. Here, we assume symmetry between choices A and B and that all agents agree on the value of a. If everyone assigns the same fixed chance a greater than 50% that a neighbor will choose the best alternative, all agents will have an agreement mechanism. For $a < 50\%$ we have contrarians [12–14] and when $a = 50\%$, observing another agent has no influence on the opinion of the agent making the observation and it could be considered an inflexible [15–17]. In principle, a could be a function of i and j. In this case, it would play the role of trust between the agents [18], measuring how much i trusts j.

With the introduction of a social network or a similar structure that specifies who can be influenced by whom, how often, and the number of agents involved, the CODA model is ready. Different approaches are possible, like majority rules, and were indeed used in variations of CODA [17]. In any case, we have two different aspects: how agents change their minds and who they interact with.

For the CODA model and variations, most of the time, a change of variable can be very useful. This model is much simpler when we work with the log-odds ν in favor of $+1$, defined as $\nu_i = \ln(\frac{p_i}{1-p_i})$. The Bayes Theorem causes a change in p_i that translates to a simple additive process in ν_i. That is, if the neighbor supports $+1$ (prefers A), ν_i changes to $\nu_i + \alpha$, where $\alpha = \ln(\frac{a}{1-a})$; if the neighbor supports -1, ν_i changes to $\nu_i - \alpha$. That is, the model is a simple additive biased random walk, with the bias dependent on the choice of the neighbors of each agent. While the model was described here as one agent influencing another one, the details of the calculations do not depend on that. We could have two agreeing agents influencing several others as in the Sznajd model [4,5,19] and the equations would be the same, with just the meaning of a altered to represent the agreement between the two speaking agents. This was actually already implemented in the paper where CODA was first defined [10]. Majority models are also defined simply by changing the rule of interaction and a version with strength of opinions as in CODA has already been studied [20]. In CODA, strength of opinion exists and opinions are continuous, unlike the discrete models. Here, the objective is to see that the rule we obtain from CODA can be expanded in a way where a discrete rule of imitation can be obtained as a limit case.

When the spatial structure is introduced, simulations have shown [10,11] that the emerging consensus is only local. Neighborhoods that support one idea will reinforce themselves and, with time, most of the agents become more and more sure of their opinions, to the point they can be described as extremists. An extremist is defined as someone who is very close to being sure about one issue (very large $|\nu_i|$, corresponding to p_i very close to certainty, 0 or 1). This happens even when all the agents had moderate opinions as initial conditions, unlike other models, where extremists have to be artificially introduced from the beginning. One should notice that the underlying continuous opinion allows us to speak of strength of opinions, unlike typical discrete models and, as such, at first, it is not so clear how CODA relates to those models.

When analyzed using the framework, it is clear how one can generalize the CODA model to different scenarios. For example, by modeling a situation where $\alpha \neq \beta$ and β is a function of time, it was possible to obtain a diffusive process from the CODA model where the diffusion slows down with time until it freezes [21], with clear applications in the spread of new ideas or products. By modeling the influence of Nature as a bias in the social process of science, CODA also proved useful to improve the understanding of how scientific knowledge might change [22].

As an extension of the model, we can assume that the likelihoods depend not only on the opinion of the neighbor, but also on the agent's own observed choice. This is equivalent to introducing in the agent some awareness that its neighbor's choices might be dependent not only on the best choices, but could also be a reflection of its own influence upon that neighbor. For calculation purposes, assume, without lack of generality, that the first agent choice is $s_i = +1$. That is, the likelihood $P(s_j = +1|x = 1)$ is replaced by two different probabilities

$$\begin{aligned} a &= P(s_j = +1|x = +1, s_i = +1) \\ &\neq P(s_j = +1|x = +1, s_i = -1) = c \end{aligned} \tag{1}$$

and $P(s_j = -1|x = -1)$ is replaced by

$$\begin{aligned} b &= P(s_j = -1|x = -1, s_i = -1) \\ &\neq P(s_j = -1|x = -1, s_i = +1) = d. \end{aligned} \tag{2}$$

Solving the Bayes Theorem and calculating the log-odds of the opinion, if the neighbor agrees ($s_i = +1$), we have

$$\nu(t+1) = \nu(t) + \ln\left(\frac{a}{1-d}\right), \tag{3}$$