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Following a trend with an exponential moving average: Analytical results for a Gaussian model

Denis S. Grebenkov [a,*], Jeremy Serror [b]

[a] Laboratoire de Physique de la Matière Condensée, CNRS – Ecole Polytechnique, 91128 Palaiseau, France

[b] John Locke Investment, 38 Avenue Franklin Roosevelt, 77210 Fontainebleau-Avon, France

HIGHLIGHTS

- How stock prices are transformed into profit-and-loss of a trend following strategy?
- We derive the probability distribution of P&L for a Gaussian model.
- We compute the net annualized risk adjusted P&L and the strategy turnover.
- We deduce the trend following optimal timescale and study its behavior.
- TF strategies admit large losses at short time but ensure small losses at longer time.

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ABSTRACT

We investigate how price variations of a stock are transformed into profits and losses (P&Ls) of a trend following strategy. In the frame of a Gaussian model, we derive the probability distribution of P&Ls and analyze its moments (mean, variance, skewness and kurtosis) and asymptotic behavior (quantiles). We show that the asymmetry of the distribution (with often small losses and less frequent but significant profits) is reminiscent to trend following strategies and less dependent on peculiarities of price variations. At short times, trend following strategies admit larger losses than one may anticipate from standard Gaussian estimates, while smaller losses are ensured at longer times. Simple explicit formulas characterizing the distribution of P&Ls illustrate the basic mechanisms of momentum trading, while general matrix representations can be applied to arbitrary Gaussian models. We also compute explicitly annualized risk adjusted P&L and strategy turnover to account for transaction costs. We deduce the trend following optimal timescale and its dependence on both auto-correlation level and transaction costs. Theoretical results are illustrated on the Dow Jones index.

1. Introduction

Systematic trading has grown as an industry in finance, allowing to take rapid trading decisions for multiple stocks [1–7]. A strategy relies on price time series in the past in order to forecast price variations in near future and update accordingly its positions. Although the market complexity, variability and stochasticity damn such forecasting to fail in nearly half cases, even a tiny excess of successful forecasts is enhanced by a very large number of trades into statistically relevant profits. Many trading strategies attempt to detect an eventual trend in price series, i.e., a sequence of positively auto-correlated price variations which may be caused, e.g., by a news release or common activity of multiple traders. From a practical point

* Corresponding author. Tel.: +33 619774327; fax: +33 169334799.
E-mail addresses: denis.grebenkov@polytechnique.edu (D.S. Grebenkov), jeremy.serror@jl-investments.com (J. Serror).

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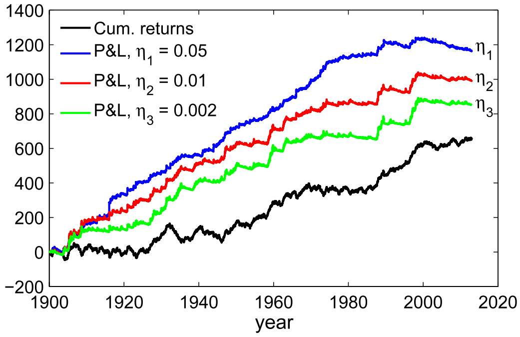

Fig. 1. Cumulative standardized logarithmic returns (normalized by realized volatility) of the Dow Jones index, from 1900 to 2012 (black curve), and cumulative P&Ls of trend following strategies (defined in Section 2) with timescales $\eta_1 = 0.05$ (blue), $\eta_2 = 0.01$ (red), and $\eta_3 = 0.002$ (green), applied to this index.

of view, a strategy transforms the known past information into a signal for buying or selling a number of shares. From a mathematical point of view, systematic trading can be seen as a transformation of price time series into profit-and-loss (P&L) time series of the strategy, as illustrated in Fig. 1. For instance, the passive (long) strategy of buying and holding a stock corresponds to the identity transformation. The choice for the optimal strategy depends on the imposed risk–reward criteria.

In this paper, we study the transformation of price variations into P&Ls of a trend following strategy based on an exponential moving average (EMA). This archetypical strategy turns out to be the basis for many systematic trading platforms [1–7], while other methods such as the detrending moving average analysis or higher-order moving averages can also be employed [8–11]. A trend following strategy is known to skew the probability distribution of P&Ls [12,13], as we illustrate in Fig. 2. This figure shows how empirically computed quantiles of price variations[1] are transformed into quantiles of P&Ls for the Dow Jones index (1900–2012). Even for such a long sample with 30 733 daily returns, accurate estimation of quantiles remains problematic. Moreover, the basic mechanisms of this transformation remain poorly understood. For these reasons, we will study a simple model in which standardized logarithmic returns are Gaussian random variables [14] whose auto-correlations reflect random trends. Even though heavy tailed asymptotic distribution of returns and some other stylized facts are ignored [15–24], the Gaussian hypothesis will allow us to derive analytical results that can be later confronted to empirical market data. We will compute the probability distribution of P&Ls of a trend following strategy in order to understand how the Gaussian distribution of price variations is transformed by systematic trading. The respective roles of the market (positive auto-correlations) and of the strategy itself, onto profits and losses, will therefore be disentangled.

The paper is organized as follows. In Section 2, we introduce matrix notations, a market model and a trend following strategy. Main results about the probability distribution and moments of P&Ls are presented in Section 3. Discussion, conclusion and perspectives are summarized in Section 4.

2. Market model and trading strategy

2.1. Exponential moving average

The exponential moving average (EMA) is broadly employed in signal processing and data analysis [25–28]. The EMA can be defined as a linear transformation of a time series $\{x_t\}$ to a smoother time series $\{\tilde{x}_t\}$ according to

$$\tilde{x}_t = \lambda \sum_{k=0}^{\infty} (1-\lambda)^k x_{t-k}, \tag{1}$$

where $0 < \lambda \leq 1$ is the (inverse of) timescale. When $\lambda = 1$, the EMA is the identity transformation: $\tilde{x}_t = x_t$; in contrast, many terms x_{t-k} effectively contribute to $\tilde{x}_t$ when $\lambda \ll 1$. The EMA is often preferred to simple moving average over a window of fixed length because it yields smoother results. In practice, it can be computed in real time according to a recurrent formula:

$$\tilde{x}_t = (1-\lambda)\tilde{x}_{t-1} + \lambda x_t.$$

When a time series starts from $t = 1$, the non-existing elements $x_0, x_{-1}, x_{-2}, \ldots$ are set to 0. This is equivalent to setting the upper limit in Eq. (1) to $t - 1$. In the analysis of a finite sample of length T, the EMA can be written in a matrix form as

$$\begin{pmatrix} \tilde{x}_1 \\ \cdots \\ \tilde{x}_T \end{pmatrix} = \lambda \mathbf{E}_{1-\lambda} \begin{pmatrix} x_1 \\ \cdots \\ x_T \end{pmatrix},$$

[1] Here, by price variations we mean cumulative standardized logarithmic returns (normalized by realized volatility), to get closer to the Gaussian hypothesis [14].