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Sheared disk packings as a model system for complex dynamics

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HIGHLIGHTS

- The soft disk model of foams is a prototypical system for complex dynamics.
- Rescaled shear stress trends are correlated to structural rearrangements of disks.
- Shear stress fluctuations display volatility clustering and long-range memory.

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ABSTRACT

Stress fluctuations in a model of densely packed disks under steady shear reproduce many features known for complex systems. These include fat-tailed probability distributions, volatility clustering and long-range autocorrelations. Using a rescaling analysis developed in econophysics, we relate stress rises and falls to rearrangements of the disks.

1. Introduction

Complex behavior of a system of interacting units (particles, agents) is often due to the non-linear interactions between these units at the local level. Here we present a study of global stress fluctuations in a dense packing of interacting soft disks under shear. This so-called bubble model or soft disk model, first developed by Durian [1,2] and later refined by Langlois et al. [3], captures many of the characteristics of two-dimensional liquid foams. In particular, it has been shown to reproduce the empirical Herschel–Bulkley relationship for the variation of the average value of the stress with strain rate for flowing foams [3–7].

In experiments and simulations of flowing foams, often such average, steady-state values are the key quantities of interest. However, the character of fluctuations can provide additional information about the system. For the soft disk model of two-dimensional flowing foams in a linear Couette geometry, Durian examined the distribution of elastic energy changes occurring due to structural rearrangements [2]. He noted avalanche behavior in the disk rearrangements, and suggested power law scaling for small, negative elastic energy changes with an exponential cutoff at large energy changes. Lauridsen

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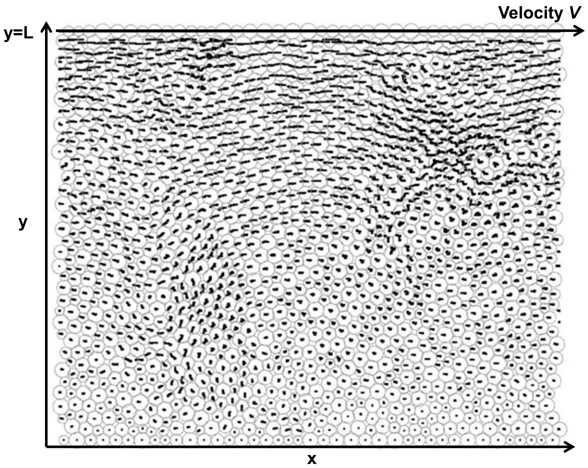

Fig. 1. Visualization of simulated system. Disks at the bottom boundary are kept fixed while disks at the top boundary are moved at constant velocity V. The resulting shear causes complex, swirling motion and non-trivial rearrangements of the disks, leading to stress fluctuations at the boundaries. The black trails represent the positions of bubble centers during a total strain of $\Delta\gamma = 0.02$.

et al. performed a similar analysis for experiments of two-dimensional flowing foams in a cylindrical Couette geometry, with similar results to Durian, but now for stress drops [8]. Dennin went on to look at the statistics of these stress drops. In particular, he measured the number of irreversible rearrangements (T1 events) in the foam and compared their occurrences to the variations of the shear stress with strain, noting a correlation between T1 events and stress changes [9].

In this study we look at the fluctuations of stress about the mean obtained for simulations using the soft disk model. Probability distribution functions of stress changes show asymmetry between rises and falls, with fat tails for stress drops. The magnitude (or volatility) of stress changes is found to exhibit clustering, suggestive of long-memory processes. Such long-range correlations are encountered in complex systems as diverse as economic markets [10,11], online betting [12], seismology [13], and internet traffic [14]. Finally, we employ a rescaling method used in the econophysics community to analyze so-called microtrends in the data, and apply this to the analysis of contact changes of the bubbles as stress is built up and released [15]. Since the local interactions are well defined, the soft disk model may be seen as a prototypical system for complex dynamics. This adds to the canon of insights into complexity and emergent behavior that can be gleaned from foam physics [16].

2. Bubble model

We performed simulations of two-dimensional wet foams using the soft disk model [1], as implemented by Langlois et al. [3] and in previous work [4] (see the Appendix). The system is comprised of 1580 bubbles, represented by circular disks, in a disordered packing sheared at a low strain rate. Overlapping disks interact via two linear forces—a harmonic repulsion proportional to their overlap, and a viscous dissipation proportional to their relative velocities. The bubbles are confined in a rectangular geometry by two boundaries, distance L apart, with semi-periodic boundary conditions in the horizontal direction (see Fig. 1). The bubbles at the lower boundary have their positions fixed and are stationary. The bubbles at the upper boundary have their y coordinates fixed, and are given a constant velocity V in the x direction, resulting in a constant strain-rate simulation. The sums of forces acting on the boundaries in x and y directions, divided by the length of the boundaries, are the shear and normal stresses respectively. We are interested in the fluctuations of these stresses once a steady state is reached, i.e. once the strain, $\gamma = Vt/L$ where t is time, has far exceeded the yield strain (which is typically a few percent). Further details of the simulation can be found in the Appendix.

To relate structural changes in the disk packing to changes in macroscopic properties such as shear stress, we measure the number of contact changes in the system. These are calculated by generating a contact matrix, $C(t)$, for each simulation timestep. $C(t)$ is an $N \times N$ matrix, where N is the number of disks in the system. If disk i and disk j are in contact at time t, the corresponding matrix element $C_{ij}(t) = 1$. If they are not, $C_{ij}(t) = 0$. By subtracting $C(t_1)$ from $C(t_2)$, summing the elements, and dividing by 2 for double counting, we calculate the number of contact changes between times t_2 and t_1.

3. Results and discussion

In this section, we present results and analysis from our simulations.