Physica A 393 (2014) 235–243

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Hot Brownian carriers in the Langevin picture: Application to a simple model for the Gunn effect in GaAs

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HIGHLIGHTS

- Langevin equation for charged particle.
- External magnetic and electric fields.
- Effective nonequilibrium temperature.
- System evolution to a nonequilibrium steady state.

ARTICLE INFO

Article history:
Received 30 April 2013
Received in revised form 31 July 2013
Available online 9 September 2013

Keywords:
Brownian motion
Langevin equation
Dissipative dynamics
Evolution of nonequilibrium systems
Carrier transport

ABSTRACT

We consider a charged Brownian gas under the influence of external, static and uniform electric and magnetic fields, immersed in a uniform bath temperature. We obtain the solution for the associated Langevin equation, and thereafter the evolution of the nonequilibrium temperature towards a nonequilibrium (hot) steady state. We apply our results to a simple yet relevant Brownian model for carrier transport in GaAs. We obtain a negative differential conductivity regime (Gunn effect) and discuss and compare our results with the experimental results.

1. Introduction

The ubiquitous Brownian motion remains an outstanding paradigm in modern physics. Some representative, but by no means an exhaustive list of general references (“founding papers”, reviews and applications) are presented in Refs. [1–19]. Here we present the Langevin formulation for a Brownian carrier in uniform and static external fields. Some recent work on charged Brownian particles is referenced in Refs. [20–47]. In our previous work on this matter, our approach hinged on the resolution of Kramers and/or Smoluchowski equations [24,26,28,45–47], and recently we began to tackle Langevin’ s formulation of this problem [47]. Here we explore the latter, in order to study the relaxation of the Brownian carrier towards a steady state, given electrical and magnetic external static and uniform fields. In Section 2 we present the solution of Langevin’s equation including the above mentioned fields. In Section 3 we present our results for the nonequilibrium temperature relaxation to the “hot” steady state temperature (as modified by the electric and magnetic fields). The computed final “hot” regime temperature is compared to the long time existing results ([48] with no magnetic field present) and with our previous results, with the magnetic field contribution, via Kramers and Smoluchowski equations [26,28,45]. In Section 4 we present an application, namely a simple yet relevant Brownian model (with no adjustable parameters) for GaAs carrier

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0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.08.081

mobility [49–56]. The multivalley band structure, and the ''hot'' carrier steady state temperature obtained in the previous section are the essential ingredients for the appearance of a negative differential conductivity regime, in good quantitative agreement with well known experimental results. Furthermore our model incorporates the magnetic field contribution hitherto not considered. Finally, in Section 6 we present our concluding remarks and outline further work.

2. Langevin equation for a Brownian charged particle

We briefly present the Langevin formalism for a free charged Brownian particle [10–12,18,19], with mass m, and charge q immersed in an homogeneous thermal reservoir at temperature T_R. It is essentially Newton's equation for the particle with two contributing forces: the first, a systematic dissipative force Stokes like (linear in the particle's velocity) and the second a rapidly fluctuating random force,

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F}_S + \mathbf{F}^r = -\gamma\mathbf{v} + \mathbf{F}^r(t) \qquad \tau = \frac{m}{\gamma}. \tag{1}$$

The formal solution is

$$\mathbf{v}(t) = \exp\left(-\frac{t}{\tau}\right)\mathbf{v}^0 + \frac{1}{m}\int_0^t \mathrm{d}t_1 \exp\left(\frac{t_1 - t}{\tau}\right)\mathbf{F}^r(t_1) \tag{2}$$

with the initial condition $\mathbf{v}^0 = \mathbf{v}(0)$ and τ the collision time. The random force has solely statistical properties: zero average and white noise correlations, given by the averages

$$\left\langle \mathbf{F}^R(t)\right\rangle = \mathbf{0} \qquad \left\langle F_i^r(t_1)F_j^r(t_2)\right\rangle = 2\frac{m}{\tau}k_B T_R \delta_{ij}\delta(t_1 - t_2) \tag{3}$$

where the correlation strength is such that the asymptotic average kinetic energy satisfies the equipartition theorem, in thermal equilibrium with the thermal reservoir (fluctuation dissipation theorem), and given by

$$\frac{1}{2}m\left\langle \mathbf{v}^2(t\to\infty)\right\rangle = \frac{3}{2}k_B T_R = \frac{1}{2}mV_T^2. \tag{4}$$

Following Refs. [28,45–47] (and with a slightly different notation) we now consider the Brownian carrier (charged particle) under the influence of homogeneous external, time independent, electric and magnetic fields; the electric contribution is given by $\mathbf{F}_{\mathrm{elec}} = q\mathbf{E}$ and the magnetic contribution (Lorentz's velocity dependent force) $\mathbf{F}_{\mathrm{mag}} = \frac{1}{c}q\mathbf{v}\times\mathbf{B}$. The total external force is given by

$$\mathbf{F}(\mathbf{v}) = \mathbf{F}_{\mathrm{elec}} + \mathbf{F}_{\mathrm{mag}}(\mathbf{v}) = q\mathbf{E} - m\omega\times v \quad \omega = \frac{q}{mc}\mathbf{B}. \tag{5}$$

Let us define a tensorial Stokes force by adding the Lorentz contribution to the usual Stokes force, as

$$\mathbf{F}_{\mathrm{TS}} = -\gamma\mathbf{v} - m\omega\times v = -m\boldsymbol{\Lambda}^{-1}\mathbf{v} \tag{6}$$

where ω is the usual cyclotron frequency, the magneto mobility tensor is $\mathbf{M} = m^{-1}\boldsymbol{\Lambda}$ with $\boldsymbol{\Lambda}$ a tensorial collision time, that can be cast into the form (when operating over an arbitrary vector $\mathbf{V}$)

$$\boldsymbol{\Lambda}(\tau,\omega)V = \tau\frac{\mathbf{V} + \tau\mathbf{V}\times\omega + \tau^2\omega\,(\omega\cdot\mathbf{V})}{1+\tau^2\omega^2}. \tag{7}$$

In particular, notice the familiar form for the case $\mathbf{B} = B\widehat{z}$

$$\boldsymbol{\Lambda}(\tau,\omega) = \frac{\tau}{1+\tau^2\omega^2}\begin{pmatrix} 1 & \tau\omega & 0 \\ -\tau\omega & 1 & 0 \\ 0 & 0 & 1+\tau^2\omega^2 \end{pmatrix}. \tag{8}$$

By defining such a tensorial Stokes force, Langevin's equation now reads

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -m\boldsymbol{\Lambda}^{-1}\mathbf{v} + q\mathbf{E} + \mathbf{F}^r(t) \tag{9}$$

with formal solution [57–60]

$$\mathbf{v}(t) = \exp\left(-\boldsymbol{\Lambda}^{-1}t\right)\mathbf{v}^0 + \boldsymbol{\Lambda}\left(1 - \exp\left(-\boldsymbol{\Lambda}^{-1}t\right)\right)\frac{q\mathbf{E}}{m} + \frac{1}{m}\int_0^t \mathrm{d}t_1 \exp\left(\boldsymbol{\Lambda}^{-1}(t_1 - t)\right)\mathbf{F}^r(t_1). \tag{10}$$

Using Cayley–Hamilton theorem, and Putzer [59] and Apostol [60] results, after a lengthy but straight forward calculation we obtain

$$\exp\left(\boldsymbol{\Lambda}^{-1}t\right) = a_0(t) + a_1(t)\boldsymbol{\Lambda}^{-1} + a_2(t)\boldsymbol{\Lambda}^{-2} \tag{11}$$

$$= \exp\left(\frac{t}{\tau}\right)\left(1 + \frac{1}{\omega^2}(\boldsymbol{\Lambda}^{-1} - \tau^{-1})^2(1-\cos\omega t) + \frac{1}{\omega}(\boldsymbol{\Lambda}^{-1} - \tau^{-1})\sin\omega t\right). \tag{12}$$