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A new lattice Boltzmann model for solving the coupled viscous Burgers' equation

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HIGHLIGHTS

- We propose a new lattice Boltzmann model to study the coupled viscous Burgers' equation.
- The efficiency and numerical accuracy of the proposed scheme are validated through several numerical experiments.
- The proposed scheme has higher accuracy compared with the numerical solutions reported in previous studies.

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ABSTRACT

In this paper, a new lattice Boltzmann model for the coupled nonlinear system of viscous Burgers' equation is proposed by using the double evolutionary equations. Through selecting equilibrium distribution functions and amending functions properly, the governing evolution system can be recovered correctly according to our proposed scheme, in which the Chapman–Enskog expansion is employed. The effects of space and time resolutions on the accuracy and stability of the model are numerically investigated in detail. The numerical solutions for various initial and boundary conditions are calculated and validated against analytic solutions or other numerical solutions reported in previous studies. It is found that the numerical results agree well with the analytic solutions, which indicates the potential of the present algorithm for solving the coupled nonlinear system of viscous Burgers' equation.

1. Introduction

Nonlinear wave phenomena frequently appear in many domains of mathematical physics and engineering fields, including fluid dynamics, plasma physics, quantum field theory, mathematical biology, nonlinear wave propagation, nonlinear fiber optics, etc. These nonlinear wave phenomena can be described by nonlinear partial differential equations (NPDEs). NPDEs have become a useful tool for describing these natural phenomena of science and engineering models [1,2]. Some of the most interesting features or physical systems are hidden in their nonlinear behavior and can only be studied with an appropriate method designed to tackle nonlinear problems. Because of the complexity of the nonlinear wave equations, there is no unified method to find all solutions of the nonlinear wave equations. Therefore finding accurate and efficient methods for solving the nonlinear wave equations has been an attractive research undertaking. In the past several decades, various methods for obtaining analytical and numerical solutions have attracted great attention of researchers in these fields, such as the decomposition method, the finite difference (FD) method, finite element (FE) method, variational iteration (VI) method, boundary elements (BE) method, and finite volume (FV) method, etc. [3].

In this paper, we consider the coupled nonlinear system of the viscous Burgers' equation which has been studied by Esipov [4,5]. The equations describe the propagation of shallow water waves, with different dispersion relations. The coupled

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nonlinear system of viscous Burgers' equation are described by the following NPDEs:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} + \eta u \dfrac{\partial u}{\partial x} + \alpha \dfrac{\partial (uv)}{\partial x} = 0, & x \in [a, b],\ t \in [0, T], \\ \dfrac{\partial v}{\partial t} - \dfrac{\partial^2 v}{\partial x^2} + \xi v \dfrac{\partial v}{\partial x} + \beta \dfrac{\partial (uv)}{\partial x} = 0, & x \in [a, b],\ t \in [0, T], \end{cases} \tag{1}$$

with the initial conditions

$$\begin{cases} u(x, 0) = \varphi_1(x), & x \in [a, b], \\ v(x, 0) = \varphi_2(x), & x \in [a, b], \end{cases} \tag{2}$$

and the boundary conditions

$$\begin{cases} u(a, t) = \phi_1(t), & u(b, t) = \phi_2(t), & t \in [0, T], \\ v(a, t) = \phi_3(t), & v(b, t) = \phi_4(t), & t \in [0, T], \end{cases} \tag{3}$$

where η and ξ are real constants, α and β are arbitrary constants depending on the system parameters such as Péclet number, the Stokes velocity of particles due to gravity, and the Brownian diffusivity [6], and $\varphi_1(x)$, $\varphi_2(x)$, and $\phi_i(t)$, $(i = 1, 2, 3, 4)$ are known functions of their arguments. This coupled nonlinear system is a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids under the effect of gravity. These equations have been found to describe various kinds of phenomena such as a mathematical model of turbulence [7] and the approximate theory of flow through a shock wave traveling in viscous fluid [8]. In the past several years, several studies used various method to obtain the analytical solution, such as the decomposition method [9], the variational iteration (VI) method [10], the modified extended tanh-function (METF) method [11] and differential transformation (DT) method [12]. There are also some numerical methods to solve the coupled nonlinear system of viscous Burgers' equation, such as the Adomian–Pade technique (APT) [13], the Chebyshev spectral collocation (ChSC) method [14], the Fourier pseudospectral (FP) method [15], the cubic B-spline collocation (CBSC) method [16], and the local discontinuous Galerkin (LDG) method [17].

In recent years, the lattice Boltzmann (LB) method has been developed into an alternative numerical method to simulate nonlinear equations and the evolution of complexity systems [18–21], especially in liquid mechanics [22–25]. Unlike conventional numerical methods, which are based on the discretization of macroscopic governing equations, and unlike the molecular dynamics method, which is based on molecular representation with complicated molecule collision rules, the LB method is based on the mesoscopic kinetic equations for particle distribution functions. The basic idea is to replace the nonlinear differential equations of a macroscopic fluid dynamics by a simplified description modeled on the kinetic theory of gases. To obtain the hydrodynamic behavior, the Chapman–Enskog expansion which exploits a small mean free path approximation to describe slowly varying solutions of the underlying kinetic equations is undertaken. From a computational viewpoint, the notable advantages are geometrical flexibility, intrinsical parallelism, simplicity of programming, numerical efficiency and ease in incorporating complex boundary conditions [22]. This method has broad prospects in a variety of fields, ranging from particle suspensions [26] to incompressible flows [27], compressible flows [23,28–39], microchannel flows [40], multiphase flow [41–53], thermal flows [48,49,51–55] and combustion phenomena [56]. Recently, the LB method has become increasingly popular in applied mathematics and scientific for solving some NPDEs, including the convection diffusion equation [57,58], Burgers' equations [59,60], the KdV–Burgers equation [61], the Poisson equation [62], the wave equation [63–66], the Kuramoto–Sivashinsky equation [67], the Benjamin–Ono equation [68], the Fokker–Planck equation [69], the Dirac equation [70–73], the Burgers–Fisher equation [74,75], the Burgers–Huxley equation [76] etc.

Motivated by the success of the LB method with double evolution equations in modeling coupled NPDE systems [59,72], the target of this paper is to further develop the lattice Boltzmann method to solve the coupled viscous Burgers' equation. To implement this method, we use two evolution equations with proper amending functions. One of the purpose of the amending functions is to recover the first-order derivative $\partial(uv)/\partial x$, which is difficult to recover by the standard LB model. In the process of connecting the new lattice Boltzmann model to the macroscopic coupled nonlinear system, we choose the appropriate equilibrium distribution functions and amending functions to satisfy some restraint conditions. The accuracy of the present scheme has also been studied, and the numerical results are compared with the analytical solution or other numerical solutions reported in previous studies.

The content of this paper is as follows. Section 2 shows our new lattice Boltzmann model and derives the coupled viscous Burgers' equation by means of the present model. Numerical validation is presented in Section 3. Finally, in the last section a brief conclusion is given.

2. Lattice Boltzmann model

The lattice Boltzmann model used in this study is based on the one-dimensional, three-velocity lattice model. The directions of the discrete velocity are defined as e_i, $i = 0, 1, 2$:

$$[e_0,\ e_1,\ e_2] = [0,\ 1,\ -1]. \tag{4}$$