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Analysis of percolation behaviors of clustered networks with partial support–dependence relations

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HIGHLIGHTS

- The robustness of clustered networks with partial support–dependence relations is studied by adopting two attack strategies.
- The first order region becomes smaller as average degree or clustering coefficient increases.
- The second order region becomes larger as average degree or clustering coefficient increases.
- Clustering coefficient has a significant impact on robustness of the system for strong coupling strength.
- For weak coupling strength, clustering coefficient has little influence, especially for attacking both networks.

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ABSTRACT

We carry out a study of percolation behaviors of clustered networks with partial support–dependence relations by adopting two different attacking strategies, attacking only one network and both networks, which help to further understand real coupled networks. For two different attacking strategies we find that the system changes from a second-order phase transition to a first-order phase transition as coupling strength q increases. We also notice that the first-order region becomes smaller and the second-order region becomes larger as average degree or clustering coefficient increases. And, as the average supported degree approaches infinity, coupled clustered networks become independent and only the second-order transition is observed, which is similar to $q = 0$. Furthermore, we find that clustering coefficient has a significant impact on robustness of the system for strong coupling strength, but for weak coupling strength it has little influence, especially for attacking both networks. The study implies that we can obtain a more robust network by reducing clustering coefficient and increasing average degree for strong coupling strength. However, for weak coupling strength, a more robust network is obtained only by increasing average degree for the same support average degree. Additionally, we find that for attacking both networks the system becomes more vulnerable and difficult to defend compared to attacking only one network.

1. Introduction

The study of complex networks is a young and active area of scientific research and appears in almost every aspect of science and technology [1–17]. Robustness of networks is a very important topic in many contexts: in communication

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networks, where equipment failures may disrupt the network and prevent users from communicating; in distribution networks, breakdowns can prevent service to customers and so on [18–22]. The robustness of network structure mainly concerns failure nodes being removed to induce a topological change, the measure of network function is given by the size of the giant component (the largest connected subnetwork) and calculating the value of critical threshold analyzed by percolation theory [23–27]. In 2009, M.E.J. Newman proposed a random-graph model of a clustered network that is exactly solvable for many of its properties including component sizes, existence and size of a giant component, and percolation properties [28]. The model forms the basis for future investigations, including epidemic processes, network resilience, and dynamical systems on networks [29–33]. Then, J.C. Miller introduced a class of random clustered networks with the same preferential mixing. He found that percolation in the clustered networks reduces the component sizes and increases the epidemic threshold compared to the unclustered networks [29]. By comparing the threshold in an unclustered network with the same degree distribution and correlation structure, J.P. Gleeson et al. found that clustering increases the epidemic threshold or decreases resilience of the network to random edge deletion [33]. Previous works have been focused on single, isolated networks where no interaction with other networks is considered, i.e., the behavior of the system is independent of any other, coupled with it. Such conditions rarely occur in nature or in technology. Typically, systems are interdependent and events taking place in one are likely to affect the others [34–41]. For instance, email and e-commerce networks rely on the Internet which in turn relies on the electric grid. In biological systems, activated genes give rise to proteins some of which go back to the genetic level and activate or inhibit other genes [42–47]. Because infrastructures in our modern society are becoming increasingly interdependent, understanding how systemic robustness due to partial interdependency is affected is one of the major challenges for designing resilient infrastructures. Recently, Buldyrev et al. developed a framework, based on percolation theory, to study the robustness of interdependent networks [34]. The studies in coupled networks highlighted the vulnerability of tightly coupled infrastructures and showed the need to consider mutually dependent network properties in designing resilient systems. Parshani et al. studied a system composed from two partially interdependent networks [35]. For two interdependent Erdos–Renyi (ER) networks, their results showed that there exists a critical threshold, below which the system shows a second-order percolation transition, while above the threshold a first-order discontinuous percolation transition occurs. Zhou et al. studied percolation behavior of two interdependent scale-free (SF) networks under random failure of a $1-p$ fraction of nodes [45]. They found that coupling strength between the two networks q reduces from 1 to 0, there exist two critical coupling strengths q_1 and q_2, which separate three different regions with different behavior of the giant component as a function of p by introducing a new analytical method. Huang et al. developed an analytical method for studying how clustering within the networks of a system of interdependent networks affects the system's robustness. They found that clustering significantly increases the vulnerability of the system [48]. Shao et al. introduced the model in coupled network systems with fully multiple support–dependence relations, which can help to further understand real-life coupled network systems, where complex dependence–support relations exists [49]. For n clustered networks, Shao et al. generalized the study of clustering of a fully coupled pair of networks and studied the robustness of a partially interdependent network of networks with clustering. Their findings highlight that interdependent networks become more vulnerable by increasing clustering coefficient for two types of model of clustered networks, which are proposed by Newman and Hackett et al. respectively [50].

Since the robustness of clustered networks with partial support–dependence relations is much more complex and practical, the analysis of percolation behaviors remains challenging and meaning. Taking this into account, this paper is organized as follows: we study the cascading failures of clustered networks with partial support–dependence relations in Section 2. In Section 3, when a clustered network with partial support–dependence relations is subjected to two different ways of attack, we analyze percolation behaviors of the system. In Section 4, our conclusions and summary are given.

2. Cascading failures of clustered networks with partial support–dependence relations

The partial support–dependence relations between two networks A and B of sizes N_A and N_B are presented by unidirectional support links, which connecting the support nodes in one network and the dependent nodes in the other network. For a node of dependent nodes $(q_{AB}N_B)$ (or $(q_{BA}N_A)$) in network B (or A), we randomly choose $\widetilde{k}_A$ (or $\widetilde{k}_B$) nodes in network A (or B) to support it, where $\widetilde{k}_A$ (or $\widetilde{k}_B$) satisfies support degree distribution $\widetilde{P}^A(\widetilde{k}_A)$ (or $\widetilde{P}^B(\widetilde{k}_B)$). We assume a functional node of dependent nodes within one network should satisfy both of the following conditions: (i) must have at least one functional support node in other networks and (ii) must belong to the giant component of functional nodes in the network it belongs to [49]. When studying cascading failure dynamics between two networks, we assume that all their support nodes in network B which are found to be functional at the previous $(t-1)$ step are still functional for nodes in network A at step t, while all their support nodes in network A which are found to be functional at the current t stage are still functional for treating nodes in network B at stage t [49]. Then, when initially a $1-p_A$ and $1-p_B$ fraction of nodes are randomly removed from both networks, the probability that the node in network A at stage t has no functional support nodes in network B is

$$\beta_t^{BA} = q_{BA}\sum_{\widetilde{k}_{BA}=0}^{\infty}\widetilde{P}^{BA}(\widetilde{k}_{BA})(1-p_{t-1}^{(B)})^{\widetilde{k}_{BA}} = q_{BA}\widetilde{G}^{BA}(1-p_{t-1}^{(B)}), \tag{1}$$