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Quantum correlations and quantum phase transition in the Ising model with Dzyaloshinskii–Moriya interaction

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HIGHLIGHTS

- Quantum correlations and quantum phase transition in Ising model with DM interaction are investigated.
- It shows some QC measures exhibit QPT after several iterations of renormalization.
- It shows GQD is an upper bound of the square QD (or negativity).
- QD is equal to negativity if D is small while QD is larger than negativity if D is large.

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ABSTRACT

In this paper, we investigate quantum correlations (QCs) and quantum phase transition (QPT) in the Ising model with Dzyaloshinskii–Moriya (DM) interaction by employing the quantum renormalization group method. The results show that some quantum correlation measures can effectively exhibit the quantum critical points associated with quantum phase transition after several iterations of the renormalization. The results also show the geometric quantum discord (GQD) is an upper bound of the square quantum discord (QD) (or negativity) in this spin system all along. Moreover, the nonanalytic appearance and scaling behaviors of the model are analyzed in detail. As a byproduct, we obtain that quantum discord is equal to negativity when the parameter of the DM interaction is small while quantum discord is larger than negativity when the spin–orbit has a strong coupling effect.

1. Introduction

Quantum entanglement, as the characteristic trait of quantum mechanics [1], has widely used in the quantum information theory (QIT) [2]. However, people have found that entanglement should not be regarded as the unique measure of quantum correlations since there exists other types of nonclassical correlations even in some mixed separable (not entangled) states [3–7]. Among them, quantum discord firstly introduced by Ollivier and Zurek [6] is the most useful and valuable method to characterize the nonclassical correlations in the two-qubit states. It has been proven that even entanglement vanishes there still exists quantum discord even in some mixed separable (not entangled) states [8–12]. It is very interesting and essential to study the dynamic behaviors of quantum discord. Therefore, the dynamics of quantum discord has attracted considerable attention in recent years [13–21].

In particular, quantum spin systems, as one of the important physical systems to research the affluent properties of quantum correlations, can be used to construct a quantum computer. Meanwhile, Dzyaloshinskii–Moriya interaction, both the Ising model in transverse field and anisotropic spin XXZ models can be added up a magnetic term, arises from the

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spin–orbit coupling, and it can restore the spoiled entanglement via creation of the quantum fluctuation. Several schemes concerning the effect of Dzyaloshinskii–Moriya (DM) interaction on the quantum discord as well as entanglement in the spin-chain systems has emerged [22–27]. Authors [22] have investigated the role of DM interaction in the ground state and thermal entanglement of a Heisenberg XYZ two-qubit system in the presence of an inhomogeneous magnetic field. It has been demonstrated that the entanglement of ground state tends to vanish asymptotically for a certain value of DM parameter D, and the entanglement undergoes a revival when D crosses its critical value. Recently, Ma et al. [26] have studied the influence of DM interaction on the quantum discord and the thermal entanglement of a spin star model, and it has been illustrated that the strong DM interaction can enhance the quantum discord and thermal entanglement while the external magnetic field with a large value and a high temperature suppresses them.

Furthermore, quantum phase transition (QPT) in spin-chain systems is also a significant research topic in condensed matter physics and statistical physics. QPT, which is characterized by detecting the nonanalytic behaviors near the critical point in some physical traits of the systems, is induced by the change of an external parameter or coupling constant [28]. This change occurs at absolute zero temperature where the quantum fluctuations play a dominant role while all the thermal fluctuations become frozen [29–33]. People have found that QPT is closely related to the entanglement of ground state in the spin system. When the QPT happens, the entanglement of ground state always produces an abrupt change [34–40]. However, employing the quantum renormalization-group (QRG) method, which is explicitly designed to analyze statistical continuum limit problems, to investigate the performance of quantum correlations in QPT of the Ising model with Dzyaloshinskii–Moriya interaction seems to be seldom investigated before. Motivated by this, in this paper, our main idea is to use the QRG method to study dynamical behavior of renormalization of various quantum correlation measures (including entanglement, quantum discord, and geometric quantum discord) in QPT. To gain a further insight, a comparison among the three quantum correlations in this model has been discussed in detail.

The paper is organized as follows. In Section 2 we will first briefly review the quantum renormalization group method in the Ising model with DM interaction. In Section 3 the dynamical behaviors of several quantum correlation measures in the quantum phase transition are investigated explicitly. In Section 4, we are devoted to studying the nonanalytic and the scaling behaviors of the model. Finally discussions are given in Section 5.

2. Quantum renormalization group

In this section, we recall the quantum renormalization group method in the Ising model with Dzyaloshinskii–Moriya interaction. Actually, the main idea of the QRG method is to explore a recursive procedure in order to cut back the degrees of freedom by reducing a certain amount of variables until reaching a fixed point. Following Kadanoff's approach, the lattice is split into blocks. Each block is treated independently to obtain the lower energy renormalized Hilbert subspace. After the above procedures, an effective Hamiltonian H^{eff} is achieved by the full Hamiltonian being mapped into the renormalized space. Now, we explicitly introduce the quantum renormalization group in the Ising model with Dzyaloshinskii–Moriya interaction. The Hamiltonian of the model in the z direction on a periodic chain of N site is

$$H = \frac{J}{4}\sum_{i}^{N}\left[\sigma_i^z\sigma_{i+1}^z + D\left(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x\right)\right] \tag{1}$$

in which J is the exchange coupling constant and $J > 0$ corresponds with the antiferromagnetic case as well as $J < 0$ ferromagnetic case; D is the strength of z component of DM interaction with $D \geq 0$. σ_i^χ $(\chi = x, y, z)$ are the standard Pauli operators at site i.

In order to get a self-similar Hamiltonian after each QRG step, it is essential for us to choose a decomposition of three-site blocks. An odd site Hamiltonian has two degenerate ground states as follows:

$$|\phi_0\rangle = \frac{1}{\sqrt{2q(1+q)}}(2D\,|{\downarrow\uparrow\uparrow}\rangle + i(1+q)\,|{\uparrow\downarrow\uparrow}\rangle - 2D\,|{\uparrow\uparrow\downarrow}\rangle) \tag{2}$$

$$\left|\phi_0'\right\rangle = \frac{1}{\sqrt{2q(1+q)}}(2D\,|{\downarrow\downarrow\uparrow}\rangle + i(1+q)\,|{\downarrow\uparrow\downarrow}\rangle - 2D\,|{\uparrow\downarrow\downarrow}\rangle) \tag{3}$$

where $|{\uparrow}\rangle, |{\downarrow}\rangle$ are the eigenstates of the σ^z Pauli matrix and

$$q = \sqrt{8D^2+1}. \tag{4}$$

Then the effective Hamiltonian of the renormalized Ising chain with DM interaction can be written as:

$$H^{eff} = \frac{J'}{4}\sum_{i}^{N}\left[\sigma_i^z\sigma_{i+1}^z - D'\left(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x\right)\right] \tag{5}$$

where the iterative relationship is

$$J' = J\left(\frac{1+q}{2q}\right)^2, \qquad D' = \frac{16D^3}{(1+q)^2}. \tag{6}$$