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Fisher–Shannon product and quantum revivals in wavepacket dynamics

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HIGHLIGHTS

- Fisher–Shannon product is used to study dynamics of quantum wave packets.
- The collapse and revival sequences in the dynamics are described.
- We have studied two models: a quantum bouncer and a graphene quantum ring.

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ABSTRACT

We show the usefulness of the Fisher–Shannon information product in the study of the sequence of collapses and revivals that take place along the time evolution of quantum wavepackets. This fact is illustrated in two models, a quantum bouncer and a graphene quantum ring.

1. Introduction

Sequences of collapses and revivals in wavepacket temporal evolution are a well known aspect of quantum dynamics. This phenomenon has been theoretically understood [1] and to date it has been observed in striking experiments with atoms and molecules [2,3], Bose–Einstein condensates [4,5] and recently in coherent states in a Kerr medium [6]. Moreover, quantum revivals have been studied theoretically in low-dimensional quantum structures such as graphene, graphene quantum dots and rings in perpendicular magnetic fields [7–15].

In this paper we show that the analysis of wavepacket quantum revivals can be carried out using the Fisher–Shannon product P_ρ, defined as [16]:

$$P_\rho = I_\rho N_\rho \tag{1}$$

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with

$$I_\rho = \int \frac{|\nabla\rho(x)|^2}{\rho(x)}\mathrm{d}x \tag{2}$$

being the Fisher information,

$$S_\rho = -\int \rho(x)\ln\rho(x)\mathrm{d}x \tag{3}$$

the Shannon entropy, and $N_\rho = \exp(2S_\rho)/(2\pi e)$ the so-called entropy power [17]. It is known that these information measures show complementary descriptions of the spreading or concentration of the probability density, where the Fisher information gives a local measure of the spreading (due to the gradient in the functional form), whereas the Shannon entropy provides a global one. This entropic product has proved useful in the analysis of different physical situations, i.e., electronic correlation [16], atomic physics [18–20], chemical reactions [21], quantum phase transitions [22], astrophysics [23] or in the study of geophysical phenomena [24]. There is a generalization of the Fisher–Shannon product, the so-called Fisher–Rényi product [25]. Note here the existence of other important complexity measures which have also been used in the description of a great variety of systems (see Ref. [26] and references therein).

We shall consider the Fisher–Shannon information product as it applies to quantum revival phenomena. In particular, we shall show the role of this quantity in the dynamics of two model systems that exhibit sequences of quantum collapses and revivals: a so-called quantum 'bouncer' (that is a quantum particle bouncing against a hard surface under the influence of gravity) and a graphene quantum ring model.

2. Wavepacket dynamic and Fisher–Shannon product

It is well known that the temporal evolution of localized bound states ψ for a time independent Hamiltonian is given in terms of the eigenvectors u_n and eigenvalues E_n as

$$\psi(t) = \sum_{n=0} c_n u_n \mathrm{e}^{-iE_n t/\hbar}, \tag{4}$$

where $c_n = \langle u_n, \psi\rangle$ are the Fourier components of the vector ψ, and n is the main quantum number of the system (in general one has to consider the set of quantum numbers corresponding to the system, see Ref. [1], but in this paper we will consider only systems with one quantum number). Now, a wavepacket is constructed with the coefficients c_n tightly centered around a large value of $n_0 \gg |n - n_0|$, with $n_0 \gg 1$. The exponential factor in (4) can then be written as a Taylor expansion around n_0 (within this approximation, n is a continuous variable) as

$$\begin{aligned}\exp(-iE_n t/\hbar) &= \exp\left[-i(E_{n_0} + E'_{n_0}(n-n_0) + E''_{n_0}/2(n-n_0)^2 + \cdots)t/\hbar\right]\\ &= \exp\left(-i\omega_0 t - 2\pi i(n-n_0)t/T_{\mathrm{Cl}} - 2\pi i(n-n_0)^2 t/T_{\mathrm{R}} + \cdots\right)\end{aligned} \tag{5}$$

where each term in the exponential (except for the first one, which is a global phase) defines a characteristic timescale, that is, $T_{\mathrm{R}} \equiv \frac{4\pi\hbar}{|E''_{n_0}|}$ and $T_{\mathrm{Cl}} \equiv \frac{2\pi\hbar}{|E'_{n_0}|}$ (see Ref. [1] for more details). The so called fractional revival times can be given in terms of the quantum revival timescale by $t = pT_{\mathrm{R}}/q$, where p and q are mutually prime.

Next, we study the wavepacket dynamics by means of the so-called entropy product, i.e., the product of the Fisher information and the Shannon entropy power, N_ρ, to conclude that it provides another framework for visualizing fractional revival phenomena. Again, we expect that the formation of a number of minipackets of the original packet will correspond to relative minima of the information product. Before proceeding, recall that P_ρ satisfies the isoperimetric inequality [17]

$$P_\rho = I_\rho N_\rho \geq 1. \tag{6}$$

The equality is reached for Gaussian densities. Combining the above inequality with the Stam uncertainty principle [27]

$$I_\rho \geq \frac{4}{\hbar^2}(\Delta p)^2, \tag{7}$$

and the power entropy inequality [28]

$$N_\rho \leq (\Delta x)^2, \tag{8}$$

leads to the usual formulation of the uncertainty principle in terms of the variance in conjugate spaces, $\Delta p\Delta x \geq \hbar/2$. It is straightforward to show that the equality limit of these four inequalities is reached for Gaussian densities.